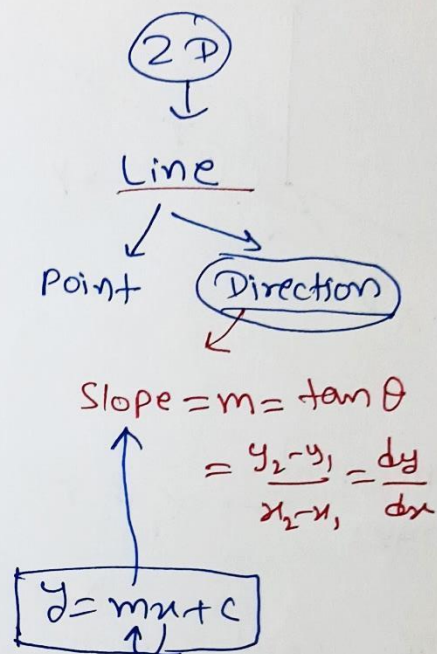
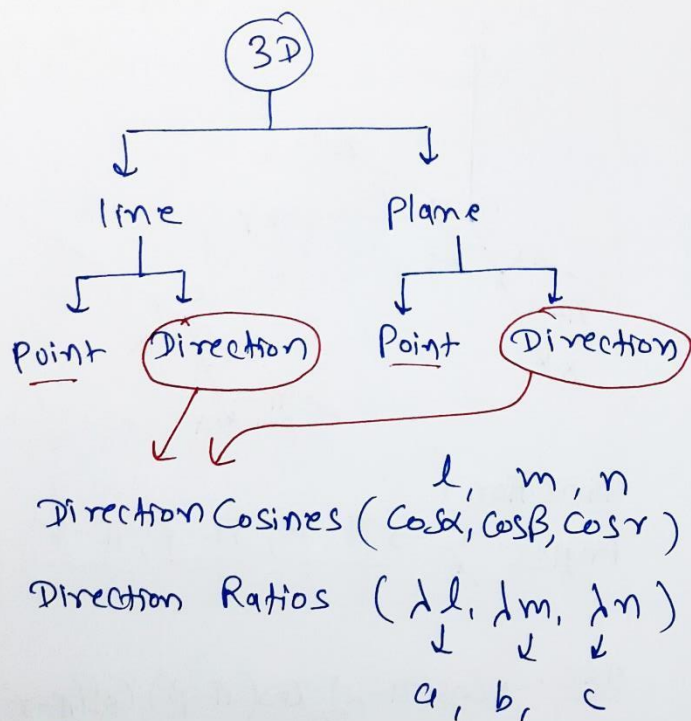


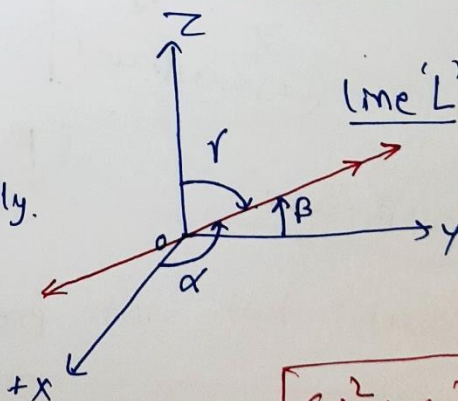
Direction Cosines (D.C.) & Direction Ratios (D.R.)

(दिक्कोण और दिक् अनुपात)

Direction Cosines (D.C.) & Direction Ratios (D.R.)

$\alpha, \beta, \gamma \rightarrow$ Direction Angles
 between Line 'L'
 and OX, OY, OZ respectively.

$(+x) (+y) (+z)$



* $(\cos \alpha, \cos \beta, \cos \gamma)$
 $\downarrow \quad \downarrow \quad \downarrow$
 $l \quad m \quad n$ → Direction Cosines

$(\lambda \cos \alpha, \lambda \cos \beta, \lambda \cos \gamma)$
 $\lambda l, \lambda m, \lambda n$ → Direction Ratios
 $(\text{Proportional to the D.C.})$
 a, b, c

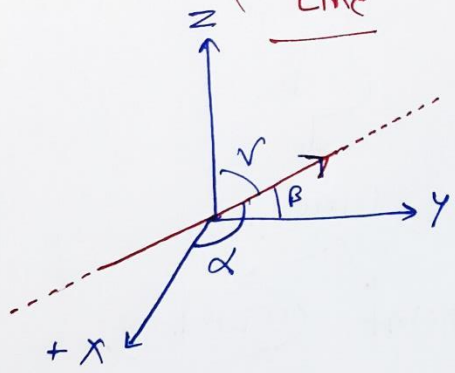
$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$l^2 + m^2 + n^2 = 1$$

$$\lambda \in \mathbb{R} - \{0\}$$

Note: There are two possibilities of Direction Cosines (D.C.) for the same line.

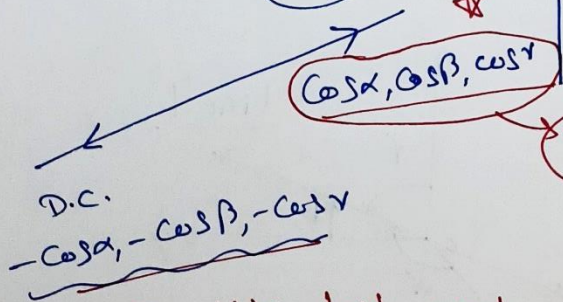
(Given Directed Line)



Direction Angles $\rightarrow \alpha, \beta, \gamma$

D.C. $\cos \alpha, \cos \beta, \cos \gamma$

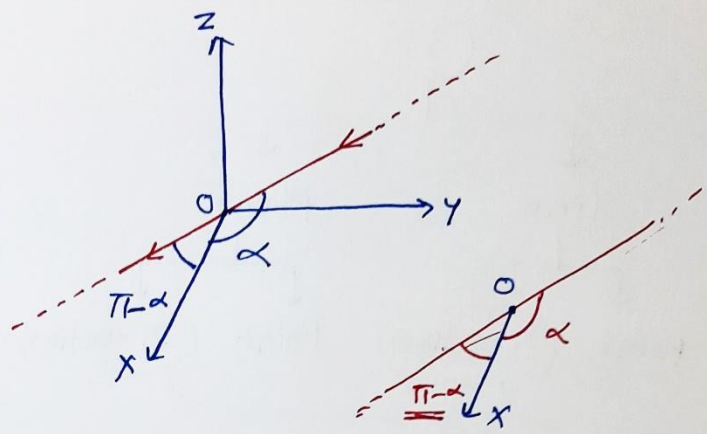
(Given)



Preferred

possible but not preferred

(Reversed Line)



Direction Angles $\rightarrow \pi - \alpha, \pi - \beta, \pi - \gamma$

D.C. $\rightarrow \cos(\pi - \alpha), \cos(\pi - \beta), \cos(\pi - \gamma)$
 $\rightarrow \underline{-\cos \alpha, -\cos \beta, -\cos \gamma}$

Conversion

D.C. $\rightsquigarrow$ D.R.

$$l, m, n \rightarrow \lambda l, \lambda m, \lambda n$$

$$\cos \alpha, \cos \beta, \cos \gamma \rightarrow \lambda \cos \alpha, \lambda \cos \beta, \lambda \cos \gamma$$

$$\lambda \in \mathbb{R} - \{0\}$$

D.R.

$$\left\{ \begin{array}{l} \lambda = 1, \quad \underline{l, m, n} \\ \lambda = 2, \quad \underline{2l, 2m, 2n} \\ \lambda = 3, \quad \underline{3l, 3m, 3n} \\ \vdots \quad \quad \quad \vdots \end{array} \right\}$$

D.R. $\rightsquigarrow$ D.C.

$$a, b, c \rightarrow \frac{\pm a}{\sqrt{a^2 + b^2 + c^2}}, \frac{\pm b}{\sqrt{a^2 + b^2 + c^2}}, \frac{\pm c}{\sqrt{a^2 + b^2 + c^2}}$$

Note: A line Always has

unique D.C.

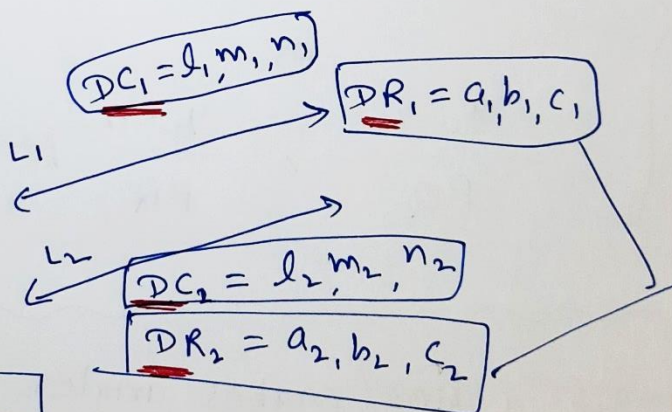
Infinite number of D.R.

Note: Parallel lines:

$$L_1 \parallel L_2$$

① $DC_1 = DC_2$ Equal

$l_1 = l_2$	$m_1 = m_2$	$n_1 = n_2$
-------------	-------------	-------------



② $DR_1 \propto DR_2$

(Proportion)

↓
(Ratio $\rightarrow$ equal)

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

or

$$a_1 : b_1 : c_1 = a_2 : b_2 : c_2$$

D.R. & D.C. of a line joining two Points.

$P(x_1, y_1, z_1)$ $Q(x_2, y_2, z_2)$

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

D.R.

$\rightarrow \begin{cases} x_2 - x_1, y_2 - y_1, z_2 - z_1 \\ \text{or} \\ K(x_2 - x_1), K(y_2 - y_1), K(z_2 - z_1) \end{cases}$

D.C.

$\left\{ \frac{(x_2 - x_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}, \frac{(y_2 - y_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}, \frac{(z_2 - z_1)}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}} \right\}$

or

$\frac{x_2 - x_1}{PQ}, \frac{y_2 - y_1}{PQ}, \frac{z_2 - z_1}{PQ}$

e.g. If a line makes $\overbrace{90^\circ, 60^\circ, 30^\circ}$ with the positive direction of x, y & z-axis respectively, find its

Direction Cosines.

$$\begin{aligned} \rightarrow D.C. &= \cos \alpha, \cos \beta, \cos \gamma \\ &= \cos 90^\circ, \cos 60^\circ, \cos 30^\circ \\ &= 0, \frac{1}{2}, \frac{\sqrt{3}}{2} \end{aligned}$$

$$\begin{array}{ccc} \alpha, & \beta, & \gamma \\ \downarrow & \downarrow & \downarrow \\ 90^\circ & 60^\circ & 30^\circ \end{array}$$

e.g. If a line has direction ratios 2, -1, -2, determine its direction cosines.

$$\boxed{D.R. = 2, -1, -2}$$

$$\begin{aligned} D.C. &= \frac{2}{\sqrt{(2)^2 + (-1)^2 + (-2)^2}}, \frac{-1}{\sqrt{4+1+4}}, \frac{-2}{\sqrt{4+1+4}} \\ D.C. &= \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3} \end{aligned}$$

e.g. Find the direction cosines of the line passing through the two points $(-2, 4, -5)$ & $(1, 2, 3)$

$$PQ = \sqrt{3^2 + (-2)^2 + 8^2}$$

$$= \sqrt{9 + 4 + 64}$$

$$= \sqrt{77}$$

$$D.R. = x_2 - x_1, y_2 - y_1, z_2 - z_1$$

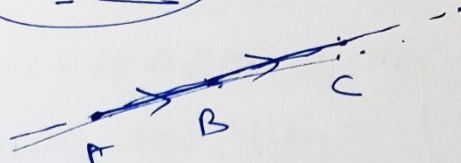
$$= \underline{\underline{3, -2, 8}}$$

$$D.C. = \frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$$

e.g. Show that the points $A(2, 3, -4)$, $B(1, -2, 3)$ & $C(3, 8, -11)$ are collinear.

in a line

$DC \& DP$



$AB + BC = CA$

D.R. $AB = ?$

D.R. $BC = ?$

Proportion
(Ratio equal)

D.R. $AB = 1-2, -2-3, 3-(-4) = -1, -5, 7$
 $(2, 3, -4) \rightarrow (1, -2, 3)$

$-1 : -5 : 7$

$1 : 5 : -7$

D.R. $BC = 3-1, 8-(-2), -11-3 = 2, 10, -14$
 $(1, -2, 3) \rightarrow (3, 8, -11)$

$2 : 10 : -14$

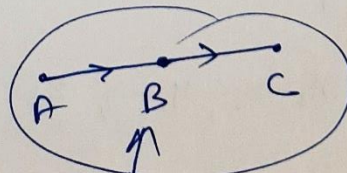
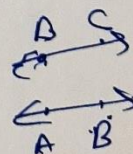
$1 : 5 : -7$

$\frac{-1}{2} = \frac{-5}{10} = \frac{7}{-14}$

$\frac{-3}{2} = \frac{-1}{2} = \frac{1}{2}$

Collinear

$AB \parallel BC$



Common Point

Exercise - 11.1

D.C. ★
D.R.

Q.1 If a line makes angles 90° , 135° , 45° with the x , y and z -axes respectively, find its direction cosines.

Direction Angles

 α, β, γ
 $\downarrow \quad \downarrow \quad \downarrow$
 $90^\circ, 135^\circ, 45^\circ$

Direction Cosines

$$\begin{aligned} & \cos \alpha, \cos \beta, \cos \gamma \\ &= \cos(90^\circ), \cos(135^\circ), \cos(45^\circ) \\ &= \left(0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \end{aligned}$$

Q.2 Find the direction cosines of a line which makes equal angles with the coordinate axes.

$$\alpha = \beta = \gamma \quad \text{D.C.} \cdot [\cos \alpha, \cos \beta, \cos \gamma] \rightsquigarrow \cos \alpha, \cos \alpha, \cos \alpha$$

Properties.

$$\Rightarrow \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\Rightarrow \cos^2 \alpha + \cos^2 \alpha + \cos^2 \alpha = 1$$

$$3 \cos^2 \alpha = 1 \Rightarrow \cos^2 \alpha = \frac{1}{3}$$

$$\Rightarrow \cos \alpha = \pm \frac{1}{\sqrt{3}}$$

$$\text{D.C.} \cdot \left[\pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}}, \pm \frac{1}{\sqrt{3}} \right]$$

Q.3 If a line has the direction ratios $-18, 12, -4$, then what are its direction cosines?

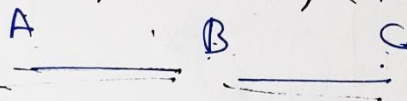
$$\text{D.R.} \rightarrow -18, 12, -4$$

$$\begin{aligned} \text{D.C.} & \rightarrow \frac{-18}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}, \frac{12}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}}, \frac{-4}{\sqrt{(-18)^2 + (12)^2 + (-4)^2}} \\ &= \frac{-18}{22}, \frac{12}{22}, \frac{-4}{22} = \left(\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11} \right) \end{aligned}$$

Points

on a line

Q.4 Show that the $(2,3,4)$, $(-1,-2,1)$, $(5,8,7)$ are collinear.



DR (Direction Ratios)

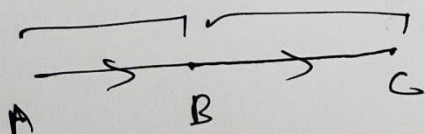
$$DR_{AB} = -1-2, -2-3, 1-4 \\ = -3, -5, -3$$

$$DR_{BC} = 5-(-1), 8-(-2), 7-1 \\ = 6, 10, 6$$

DR_{AB}	DR_{BC}
$-3, -5, -3$	$6, 10, 6$
$-3:-5:-3$	$6:10:6$
$3:5:3$	$3:5:3$

Ratio $\rightarrow$ equal.

$\text{DR} \rightarrow$ proportional $\rightarrow$ parallel



Common

$A, B, C \rightarrow$ Collinear

Method-I

$$AB + BC = AC$$

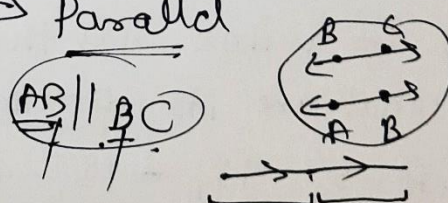
Method-II

$$\text{ar}(\triangle ABC) = 0$$

Method-III

DR, DC

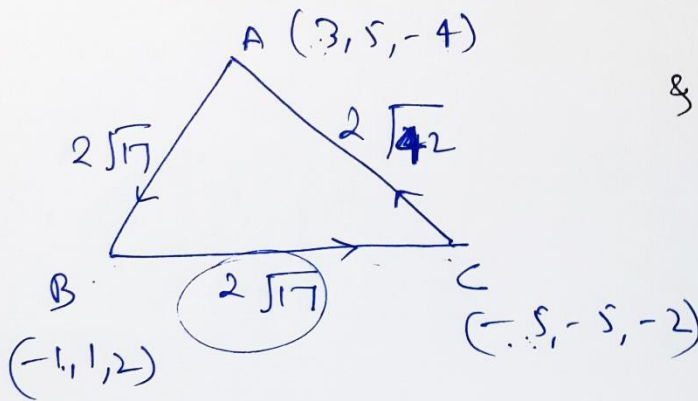
$$\frac{-3}{6} = \frac{-5}{10} = \frac{-3}{6} \\ \frac{-1}{2} = \frac{-1}{2} = \frac{-1}{2}$$



Q.5 Find the direction cosines of the sides of the triangle whose vertices are $(3, 5, -4) \rightarrow A$

$(-1, 1, 2) \rightarrow B$

& $(-5, -5, -2) \rightarrow C$



$$DC_{(AB)} = \frac{-1-3}{2\sqrt{17}}, \frac{1-5}{2\sqrt{17}}, \frac{2+4}{2\sqrt{17}}$$

$$= \frac{-4}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}, \frac{6}{2\sqrt{17}}$$

$$\rightarrow = -\frac{2}{\sqrt{17}}, -\frac{2}{\sqrt{17}}, \frac{3}{\sqrt{17}} \checkmark$$

$$DC_{BC} = \frac{-4}{2\sqrt{17}}, \frac{-6}{2\sqrt{17}}, \frac{-4}{2\sqrt{17}}$$

$$= -\frac{2}{\sqrt{17}}, -\frac{3}{\sqrt{17}}, -\frac{2}{\sqrt{17}}$$

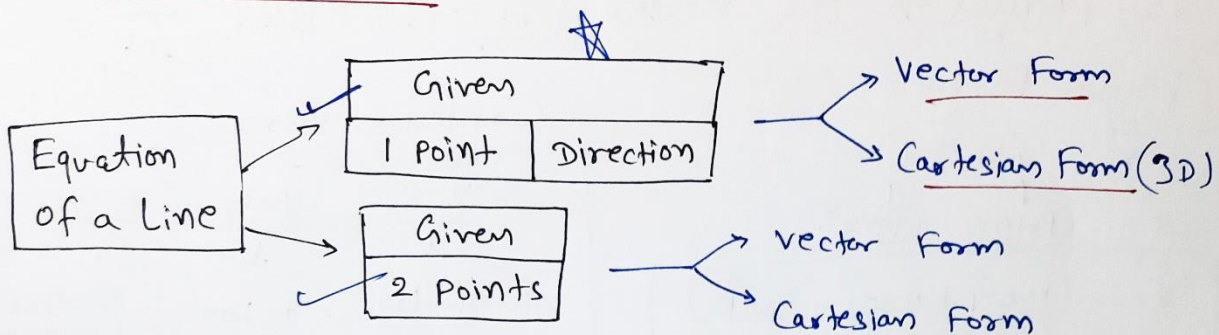
$$DC_{CA} = \frac{8}{2\sqrt{42}}, \frac{10}{2\sqrt{42}}, \frac{-2}{2\sqrt{42}}$$

$$= \frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, -\frac{1}{\sqrt{42}}$$

$$\begin{aligned} AB &= \sqrt{4^2 + 4^2 + (-6)^2} \\ &= \sqrt{68} \\ &= 2\sqrt{17} \end{aligned}$$

$$\begin{aligned} BC &= \sqrt{(-4)^2 + (-6)^2 + (-4)^2} \\ &= \sqrt{68} \\ &= 2\sqrt{17} \end{aligned}$$

$$\begin{aligned} CA &= \sqrt{8^2 + 10^2 + (-2)^2} \\ &= \sqrt{64 + 100 + 4} \\ &= \sqrt{168} \\ &= 2\sqrt{42} \end{aligned}$$

Line (in 3D)Equation of a Line (when 1 point & Direction are given)

Let the line passes through the point A & is parallel to $\vec{b}$

Coordinates of A (x_1, y_1, z_1)

Position Vector $(\vec{a})$ of A = $x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$ Point

$\vec{b} = a\hat{i} + b\hat{j} + c\hat{k}$ Direction

Equation of Line (L) $\Rightarrow$

Vector Form

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

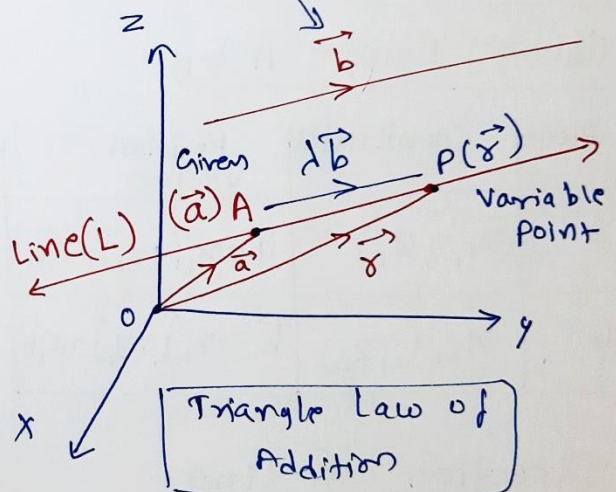
$\uparrow$ Point
 $\uparrow$ Direction

Cartesian Form

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

$\frac{a}{b}$
 $\frac{b}{c}$
 $\frac{c}{a}$

Direction Ratios



$$\vec{r} = \vec{a} + \lambda \vec{b}$$

variable Point $\downarrow$ Direction $\downarrow$

$(x\hat{i} + y\hat{j} + z\hat{k})$ $\downarrow$ $(x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$ $\downarrow$ $a\hat{i} + b\hat{j} + c\hat{k}$

Comparison

Solve

e.g. Find the vector & Cartesian form of the line passing through the point $(100, 200, 300)$ and which is parallel to the vector $7\hat{i} - 17\hat{j} + 13\hat{k}$.

Vector form,

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

↑ Point
 ↑ Direⁿ

$$\vec{r} = (100\hat{i} + 200\hat{j} + 300\hat{k}) + \lambda(7\hat{i} - 17\hat{j} + 13\hat{k})$$

Cartesian form,

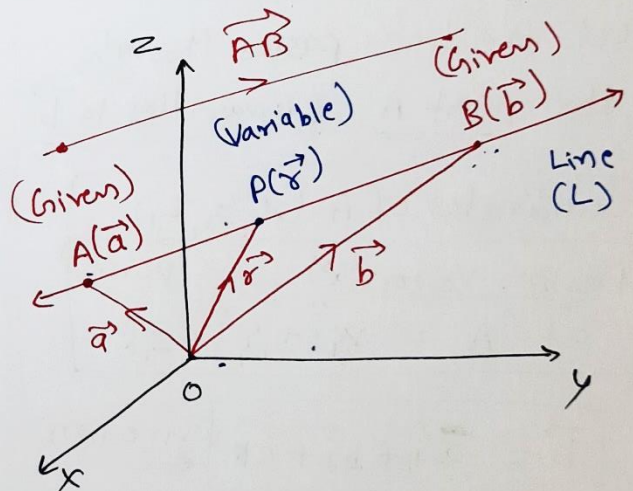
$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

$$\Rightarrow \frac{x-100}{7} = \frac{y-200}{-17} = \frac{z-300}{13}$$

Equation of a line (when two points on the line are given) →

Given Points A & B

Point	Coordinates	Position vector
A	(x_1, y_1, z_1)	$\vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$
B	(x_2, y_2, z_2)	$\vec{b} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$



Direction of line.

$$\vec{AB} = \vec{b} - \vec{a} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Equation of line(L)

Vector Form

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$

↑ Point
 ↑ Direction

Cartesian Form

$$\frac{x - \textcircled{x_1}}{\textcircled{x_2 - x_1}} = \frac{y - \textcircled{y_1}}{\textcircled{y_2 - y_1}} = \frac{z - \textcircled{z_1}}{\textcircled{z_2 - z_1}}$$

↓ Direction

e.g. Find the vector equation for the line passing through the points $(7, -17, 27)$ & $(-1, 2, -3)$.

Vector form.

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$

Point Direction

$$\vec{r} = 7\hat{i} - 17\hat{j} + 27\hat{k} + \lambda(-8\hat{i} + 19\hat{j} - 30\hat{k})$$

Cartesian form.

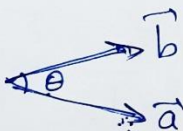
$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \frac{x-7}{-8} = \frac{y+17}{19} = \frac{z-27}{-30}$$

Angle between two lines :

Dot Product

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$



understand in Cartesian Form

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1} \quad L_1$$

$$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2} \quad L_2$$

Understand in Vector Form

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1 \quad (L_1)$$

$$\vec{r} = \vec{a}_2 + \mu \vec{b}_2 \quad (L_2)$$

Directly by Dot product.

$$\cos \theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|}$$

Conditions

Case-I perpendicular lines

$$\theta = 90^\circ$$

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

Case-II Parallel lines

$$\theta = 0^\circ$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\vec{DR}_1 \parallel \vec{DR}_2$$

e.g. Find the angle between the pair of lines given by

$$\vec{r} = \underline{3\hat{i} + 2\hat{j} - 4\hat{k}} + \lambda(\underline{\hat{i} + 2\hat{j} + 2\hat{k}}) \quad \vec{b}_1 \quad \text{Directions}$$

$$\vec{r} = \underline{5\hat{i} - 2\hat{j}} + \mu(\underline{3\hat{i} + 2\hat{j} + 6\hat{k}}) \quad \vec{b}_2$$

$$\cos\theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|}$$

$$\cos\theta = \frac{(\hat{i} + 2\hat{j} + 2\hat{k}) \cdot (3\hat{i} + 2\hat{j} + 6\hat{k})}{\sqrt{1^2 + 2^2 + 2^2} \sqrt{3^2 + 2^2 + 6^2}} = \frac{19}{21}$$

$$\theta = \cos^{-1}\left(\frac{19}{21}\right)$$

e.g. Find the angle between the pair of lines

$$\frac{x+3}{\underline{3}} = \frac{y-1}{\underline{5}} = \frac{z+3}{\underline{4}} \quad \& \quad \frac{x+1}{\underline{1}} = \frac{y-4}{\underline{1}} = \frac{z-5}{\underline{2}}$$

Directions

$$\cos\theta = \frac{(3 \times 1) + (5 \times 1) + (4 \times 2)}{\sqrt{3^2 + 5^2 + 4^2} \cdot \sqrt{1^2 + 1^2 + 2^2}} = \frac{3 + 5 + 8}{\sqrt{50} \cdot \sqrt{6}}$$

$$\cos\theta = \frac{16}{\sqrt{300}} = \frac{16}{\sqrt{100 \times 3}} = \frac{16}{10\sqrt{3}} = \frac{8}{5\sqrt{3}}$$

~~$$\theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$$~~

$$\theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$$

Distance between Two Skew Lines

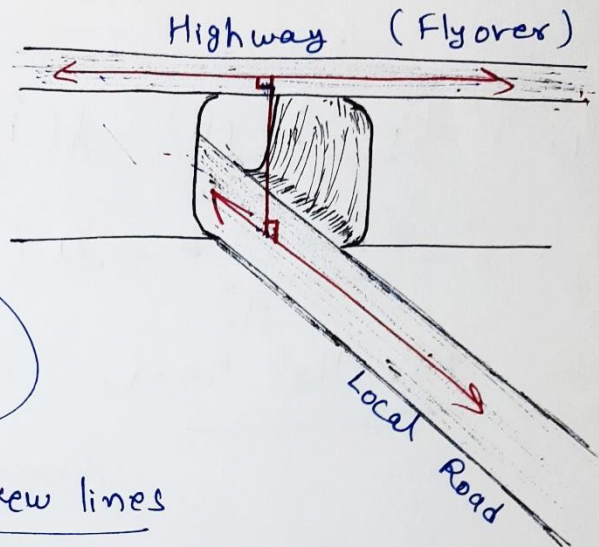
Non Coplanar
(एक plane में हो ही नहीं सकती)

not Parallel
Never meet

Skew Lines

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1 \quad (L_1)$$

$$\vec{r} = \vec{a}_2 + \mu \vec{b}_2 \quad (L_2)$$



★ Shortest Distance b/w Skew lines

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

← modulus
↑ magnitude.

Remember only
one formula
(vector form only)

Distance b/w Parallel lines

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}$$
$$\vec{r} = \vec{a}_2 + \mu \vec{b}$$

$L_1 \parallel L_2$

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|} \right|$$

e.g. Find the shortest distance between the lines l_1 & l_2

given by $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} - \hat{j} + 2\hat{k}) \leftarrow l_1$

& $\vec{r} = \hat{i} + 2\hat{j} + \mu(-\hat{i} + \hat{j} - 2\hat{k}) \leftarrow l_2$

$l_1: \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k})$

$\vec{a}_1$ $\vec{b}$

$l_2: \vec{r} = (\hat{i} + 2\hat{j}) + \mu(-\hat{i} + \hat{j} - 2\hat{k})$

$\vec{a}_2$

Parallel lines.

$\mu \neq -\mu \neq \lambda_2$

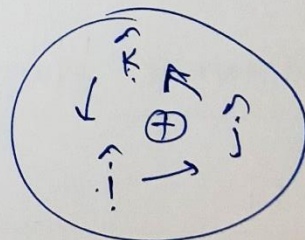
$\vec{r} = \vec{a}_1 + \lambda \vec{b}$

$\vec{r} = \vec{a}_2 + \mu \vec{b}$

d

$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \times \vec{b}}{|\vec{b}|} \right| = \left| \frac{(0\hat{i} + 0\hat{j} + 4\hat{k}) \times (\hat{i} - \hat{j} + 2\hat{k})}{\sqrt{1^2 + (-1)^2 + (2)^2}} \right|$

$d = \left| \frac{4(\hat{k} \times \hat{i}) - 4(\hat{k} \times \hat{j})}{\sqrt{1+1+4}} \right|$



$d = \left| \frac{4\hat{j} + 4\hat{i}}{\sqrt{6}} \right| = \frac{\sqrt{16+16}}{\sqrt{6}} = \frac{\sqrt{2+16}}{\sqrt{2 \times 3}}$

$d = \frac{4}{\sqrt{3}}$

e.g. Find the shortest distance between the lines

l_1 & l_2 whose vector equations are

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}) \quad \& \quad \vec{b}_1$$

$$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$$

Skew lines

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$$

$$\vec{r} = \vec{a}_2 + \mu \vec{b}_2$$

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$\vec{b}_1 \times \vec{b}_2 = (2\hat{i} - \hat{j} + \hat{k}) \times (3\hat{i} - 5\hat{j} + 2\hat{k})$$

Cross Product

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3)$$

$$\vec{b}_1 \times \vec{b}_2 = 3\hat{i} - \hat{j} - 7\hat{k}$$

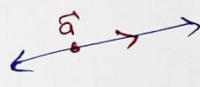
$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{9+1+49} = \sqrt{59}$$

$$d = \left| \frac{(\hat{i} - \hat{k}) \cdot (3\hat{i} - \hat{j} - 7\hat{k})}{\sqrt{59}} \right| = \left| \frac{3+7}{\sqrt{59}} \right|$$

$$d = \frac{10}{\sqrt{59}}$$

Exercise 11.2

Line (3D)



$$\vec{r} = \vec{a} + \lambda \vec{b}$$

Point

Direction.

$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$$

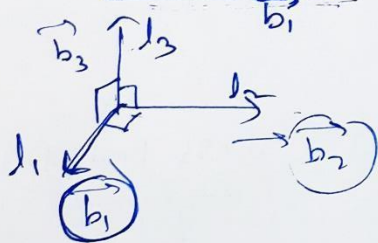
Q.1 Show that the three lines with direction cosines

$$\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13};$$

$$\frac{4}{13}, \frac{12}{13}, \frac{3}{13};$$

$$\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}$$

are mutually perpendicular.



$$\vec{b}_1 = \frac{12}{13} \hat{i} - \frac{3}{13} \hat{j} - \frac{4}{13} \hat{k}$$

$$\vec{b}_2 = \frac{4}{13} \hat{i} + \frac{12}{13} \hat{j} + \frac{3}{13} \hat{k}$$

$$\vec{b}_3 = \frac{3}{13} \hat{i} - \frac{4}{13} \hat{j} + \frac{12}{13} \hat{k}$$

$$\vec{b}_1 \cdot \vec{b}_2 = \left(\frac{12}{13} \hat{i} - \frac{3}{13} \hat{j} - \frac{4}{13} \hat{k} \right) \cdot \left(\frac{4}{13} \hat{i} + \frac{12}{13} \hat{j} + \frac{3}{13} \hat{k} \right)$$

$$= \frac{12 \times 4}{13 \times 13} - \frac{3 \times 12}{13 \times 13} - \frac{4 \times 3}{13 \times 13} = \frac{48 - 36 - 12}{13 \times 13}$$

$$= 0$$

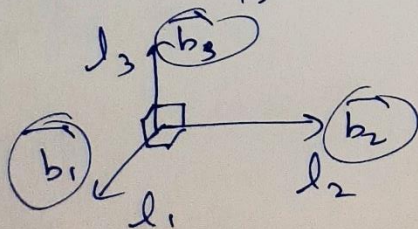
$$\vec{b}_1 \cdot \vec{b}_2 = 0$$

$$\vec{b}_1 \perp \vec{b}_2$$

$$l_1 \perp l_2$$

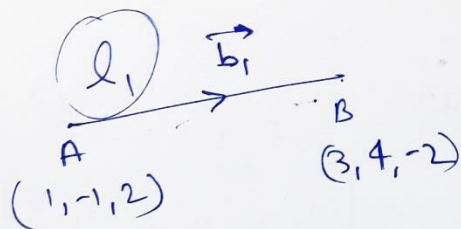
$$\vec{b}_2 \cdot \vec{b}_3 = \frac{4 \times 3}{13^2} - \frac{12 \times 4}{13^2} + \frac{3 \times 12}{13^2} = \frac{12 - 48 + 36}{13^2} = 0$$

$$\vec{b}_1 \cdot \vec{b}_3 = \frac{12 \times 3}{13^2} + \frac{3 \times 4}{13^2} - \frac{4 \times 12}{13^2} = \frac{36 + 12 - 48}{13^2} = 0$$

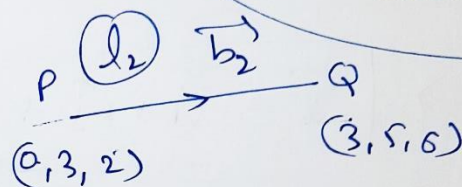
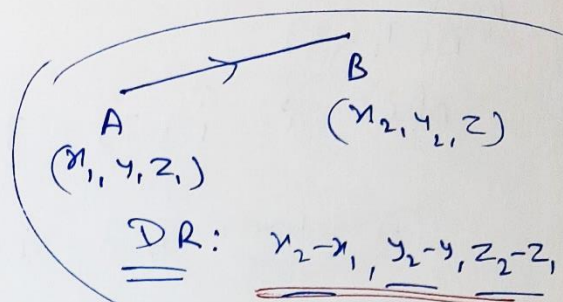


$$l_1 \perp l_2, l_2 \perp l_3, l_3 \perp l_1$$

Q.2 Show that the line through the points $(1, -1, 2)$, $(3, 4, -2)$ is perpendicular to the line through the point $(0, 3, 2)$ and $(3, 5, 6)$.

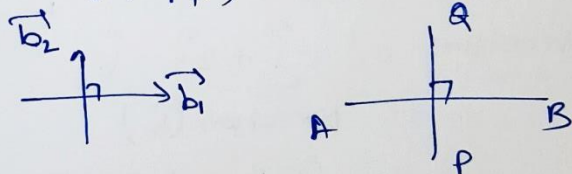


$$\begin{aligned} DR_1 &= 3-1, 4+1, -2-2 \\ &= 2, 5, -4 \\ \vec{b}_1 &= 2\hat{i} + 5\hat{j} - 4\hat{k} \end{aligned}$$

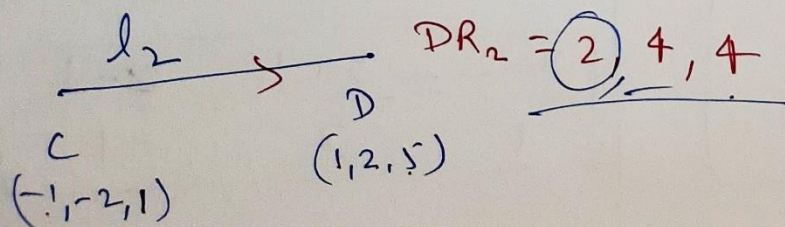
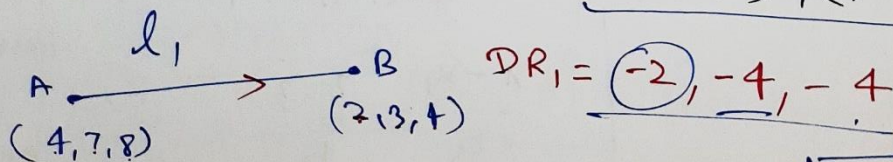


$$\begin{aligned} DR_2 &= 3, 2, 4 \\ \vec{b}_2 &= 3\hat{i} + 2\hat{j} + 4\hat{k} \end{aligned}$$

$$\begin{aligned} \vec{b}_1 \cdot \vec{b}_2 &= (2\hat{i} + 5\hat{j} - 4\hat{k}) \cdot (3\hat{i} + 2\hat{j} + 4\hat{k}) \\ &= 6 + 10 - 16 \\ &= 16 - 16 = 0 \end{aligned}$$



Q.3 Show that the line through the points $(4, 7, 8)$, $(2, 3, 4)$ is parallel to the line through the points $(-1, -2, 1)$, $(1, 2, 5)$.



$$\frac{-2}{2} = \frac{-4}{4} = \frac{-4}{4}$$

$$-1 = -1 = -1$$

DRs are proportional

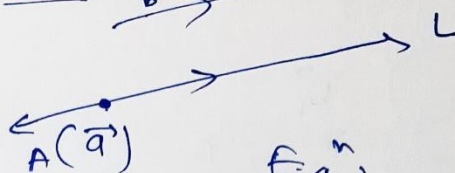
$\therefore$ lines are $\parallel$.

[Q.4] Find the Equation of the line which passes through the point $(1, 2, 3)$ and is parallel to the vector $(3\hat{i} + 2\hat{j} - 2\hat{k}) = \vec{b}$

Point
 $A(1, 2, 3)$

$$A(\vec{a}) \quad \vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

Direction, $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$



$$\vec{r} = \vec{a} + \lambda \vec{b}$$

Variation Point Direction

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$$

[Q.5] Find the Equation of the line in vector and in Cartesian form that passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in the direction of $\hat{i} + 2\hat{j} - \hat{k}$.

Point $(2, -1, 4)$

Direction $(\vec{b})$

D.R. = 1, 2, -1

Eqⁿ. of line

Vector form,

$$\vec{r} = \vec{a} + \lambda \vec{b}$$

Point Dis.

$$\vec{r} = (2\hat{i} - \hat{j} + 4\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$$

Cartesian form

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$$

Point Dir.

$$\frac{x - 2}{1} = \frac{y + 1}{2} = \frac{z - 4}{-1}$$

Q.6 Find the Cartesian eqⁿ. of the line ^{Point} ^{Dir.} which passes through the point $(-2, 4, -5)$ and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

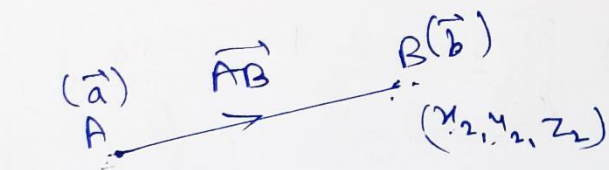
~~Dir. 3, 5, 6~~
 $(-2, 4, -5)$
 $3k, 5k, 6k$
 $\frac{x - (-2)}{3} = \frac{y - 4}{5} = \frac{z - (-5)}{6}$
 $\frac{x+2}{3} = \frac{y-4}{5} = \frac{z+5}{6}$

Q.7 The Cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$. Write its Vector Form.

$\vec{r} = \vec{a} + \lambda \vec{b}$
 Point Direction
 $\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda (3\hat{i} + 7\hat{j} + 2\hat{k})$

$(5, -4, 6)$ Point
 $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$
 Direction **DR**

Q.8 Find the Vector and the Cartesian equations of the line that passes through the origin $(0,0,0)$ and $(5,-2,3)$.



Direction = $\vec{AB}$
 $= \vec{b} - \vec{a}$

1 Point, 1 Dir.
 $\vec{r} = \vec{a} + \lambda \vec{b}$
 $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

Eqn. of line,

Vector form,

$$\vec{r} = \vec{a} + \lambda (\vec{b} - \vec{a})$$

Cartesian form,

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

ATQ $A(\vec{a}) (0,0,0)$
 $B(\vec{b}) (5,-2,3)$

Vector form

$$\vec{r} = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda (5\hat{i} - 2\hat{j} + 3\hat{k})$$

$$\vec{r} = \lambda (5\hat{i} - 2\hat{j} + 3\hat{k})$$

Cartesian form

$$\frac{x-0}{5-0} = \frac{y-0}{-2-0} = \frac{z-0}{3-0}$$

$$\Rightarrow \frac{x}{5} = \frac{y}{-2} = \frac{z}{3}$$

Q.9 Find the vector and the Cartesian Equations of the line that passes through the points

$$(3, -2, -5), (3, -2, 6).$$

A B



$$(x_1, y_1, z_1) \quad (x_2, y_2, z_2)$$

Cartesian form,

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \frac{x-3}{3-3} = \frac{y-(-2)}{-2+2} = \frac{z-(-5)}{6+5}$$

$$\Rightarrow \boxed{\frac{x-3}{0} = \frac{y+2}{0} = \frac{z+5}{11}}$$

Vector form

$$\vec{r} = \underbrace{(\vec{a})}_{\text{Point}} + \lambda(\vec{b}-\vec{a})$$

$$\Rightarrow \vec{r} = 3\hat{i} - 2\hat{j} - 5\hat{k} + \lambda((3-3)\hat{i} + (-2+2)\hat{j} + (6+5)\hat{k})$$

$$\rightarrow \vec{r} = (3\hat{i} - 2\hat{j} - 5\hat{k}) + \lambda(0\hat{i} + 0\hat{j} + 11\hat{k})$$

$$\boxed{\vec{r} = (3\hat{i} - 2\hat{j} - 5\hat{k}) + \lambda(11\hat{k})}$$

Exercise 11.2

(Angle & Distance b/w Two lines)

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$$

$$\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$$

$$\vec{r} = \vec{a}_2 + \mu \vec{b}_2$$

$$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$$

$$\cos \theta = \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1| |\vec{b}_2|}$$

$$\cos \theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Skew lines

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$$

$$\vec{r} = \vec{a}_2 + \mu \vec{b}_2$$

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

Parallel lines

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}$$

$$\vec{r} = \vec{a}_2 + \mu \vec{b}$$

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$$

Q.10 Find the angle b/w the following pairs of lines:

$$(i) \vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k}) \quad \vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k}) \quad \vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$$

$$\cos \theta = \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1| |\vec{b}_2|} = \frac{(3\hat{i} + 2\hat{j} + 6\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})}{\sqrt{9+4+36} \cdot \sqrt{1+4+4}}$$

$$\cos \theta = \frac{3+4+12}{7 \times 3} = \left(\frac{19}{21}\right) \Rightarrow \theta = \cos^{-1}\left(\frac{19}{21}\right)$$

$$(ii) \vec{r} = (3\hat{i} + \hat{j} - 2\hat{k}) + \lambda(\hat{i} - \hat{j} - 2\hat{k}) \quad | \quad \vec{r} = (2\hat{i} - \hat{j} - 5\hat{k}) + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$$

$\vec{b}_1$ $\vec{b}_2$

$$\cos \theta = \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|} = \frac{3+5+8}{\sqrt{1+1+4} \sqrt{9+25+16}}$$

$$\cos \theta = \frac{16}{\sqrt{6} \sqrt{50}} = \frac{16}{\sqrt{300}} = \frac{16}{\sqrt{100 \times 3}} = \frac{16}{10\sqrt{3}} = \frac{8}{5\sqrt{3}}$$

$$\cos \theta = \frac{8}{5\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$$

[Q.1] Find the angle between the following pair of lines:

Cartesian form

$$(i) \frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3} ; \frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$$

$(a_1) \quad (b_1) \quad (c_1) \quad (a_2) \quad (b_2) \quad (c_2)$

$$\cos \theta = \left| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \right| = \left| \frac{-2 + 40 - 12}{\sqrt{4+25+9} \sqrt{1+64+16}} \right|$$

$$\Rightarrow \cos \theta = \frac{26}{\sqrt{38} \sqrt{81}} = \frac{26}{9\sqrt{38}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

$$(ii) \frac{x}{2} = \frac{y}{2} = \frac{z}{1} \& \frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$$

$(a_1) \quad (b_1) \quad (c_1) \quad (a_2) \quad (b_2) \quad (c_2)$

$$\cos \theta = \left| \frac{8+2+8}{\sqrt{4+4+1} \sqrt{16+1+64}} \right| = \frac{18}{3 \times 8} = \frac{2}{3}$$

$$\theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Q.12 Find the value of (P) so that the lines

$$\frac{x-1}{3} = \frac{y-2}{2P} = \frac{z-3}{2} \quad \& \quad \frac{7-7x}{3P} = \frac{y-5}{1} = \frac{6-z}{5} \text{ are at Right angle.}$$

C.F.

Standard forms

$$\theta = 90^\circ$$

$$\cos 90^\circ = 0$$

$$\Rightarrow \frac{-(x-1)}{3} = \frac{7(y-2)}{2P} = \frac{z-3}{2}$$

$$\Rightarrow \frac{-7(x-1)}{3P} = \frac{y-5}{1} = \frac{-(z-6)}{5}$$

$$\Rightarrow \left[\frac{x-1}{\textcircled{-3}} = \frac{y-2}{\textcircled{\frac{2P}{7}}} = \frac{z-3}{\textcircled{2}} \right]$$

$a_1 \quad b_1 \quad c_1$

$$\Rightarrow \left[\frac{x-1}{\textcircled{\frac{3P}{-7}}} = \frac{y-5}{\textcircled{1}} = \frac{z-6}{\textcircled{-5}} \right]$$

$a_2 \quad b_2 \quad c_2$

$$\cos \theta = \left| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

$$\theta = 90^\circ$$

$$\theta = 90^\circ, \cos \theta = \cos 90^\circ = 0$$

$$\Rightarrow 0 = \left| \frac{(-3) \cdot \left(\frac{3P}{-7}\right) + \frac{2P}{7} + 2(-5)}{\sqrt{\quad} \sqrt{\quad}} \right|$$

$$\Rightarrow 0 = \frac{9P}{7} + \frac{2P}{7} - 10$$

$$\Rightarrow 10 = \frac{11P}{7}$$

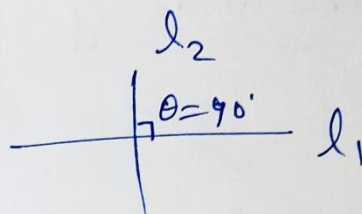
$$\Rightarrow \frac{70}{11} = P$$

Q.13

Show that the lines $\frac{x-5}{1} = \frac{y+2}{-5} = \frac{z}{1}$ & $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

$$\cos \theta = \frac{7 + (-10) + (3)}{\sqrt{49+25+1} \cdot \sqrt{1+4+9}} = 0$$

$$\cos \theta = 0 \Rightarrow \theta = 90^\circ$$



Q.14

Find the shortest distance b/w the lines

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \quad \& \quad \vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$

Point $(\vec{a}_1)$ Direction $(\vec{b}_1)$ Point $(\vec{a}_2)$ Direction $(\vec{b}_2)$

Skew lines

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$$\vec{b}_1 \times \vec{b}_2 = (\hat{i} - \hat{j} + \hat{k}) \times (2\hat{i} + \hat{j} + 2\hat{k})$$

$$d = \frac{|(\hat{i} - 3\hat{j} - 2\hat{k}) \cdot (-3\hat{i} + 3\hat{k})|}{3\sqrt{2}}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} \rightarrow \text{Expand}$$

$$\Rightarrow d = \left| \frac{-3 - 6}{3\sqrt{2}} \right| = \frac{9}{3\sqrt{2}}$$

$$= \hat{i}(-3) - \hat{j}(0) + \hat{k}(3) = -3\hat{i} + 3\hat{k}$$

$$\Rightarrow d = \frac{3}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\Rightarrow d = \frac{3\sqrt{2}}{2}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{9+9} = \sqrt{9 \times 2} = 3\sqrt{2}$$

Q.15 Find the Shortest distance b/w the lines

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

Point $\vec{a}_1 = -\hat{i} - \hat{j} - \hat{k}$ ✓

Direction $\vec{b}_1 = 7\hat{i} - 6\hat{j} + \hat{k}$ ✓

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Point $\vec{a}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k}$ ✓

Direction $\vec{b}_2 = \hat{i} - 2\hat{j} + \hat{k}$ ✓

Skew lines

$$d = \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}$$

$$d = \frac{(4\hat{i} + 6\hat{j} + 8\hat{k}) \cdot (-4\hat{i} - 6\hat{j} - 8\hat{k})}{2\sqrt{29}}$$

$$d = \frac{-16 - 36 - 64}{2\sqrt{29}}$$

$$d = \frac{+116}{2\sqrt{29}} = \frac{4 \times 29}{2\sqrt{29}} \sqrt{29}$$

$$d = 2\sqrt{29}$$

$$\vec{b}_1 \times \vec{b}_2 = (7\hat{i} - 6\hat{j} + \hat{k}) \times (\hat{i} - 2\hat{j} + \hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= \hat{i}(-4) - \hat{j}(6) + \hat{k}(-8)$$

$$\vec{b}_1 \times \vec{b}_2 = -4\hat{i} - 6\hat{j} - 8\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{16 + 36 + 64}$$

$$= \sqrt{116}$$

$$= \sqrt{4 \times 29}$$

$$= 2\sqrt{29}$$

Q.16 Find the shortest distance between the lines

whose vector equations are

$$\vec{r} = \underbrace{(\hat{i} + 2\hat{j} + 3\hat{k})}_{\text{Point } (\vec{a}_1)} + \lambda \underbrace{(\hat{i} - 3\hat{j} + 2\hat{k})}_{\text{Direction } (\vec{b}_1)}, \quad \vec{r} = \underbrace{(4\hat{i} + 5\hat{j} + 6\hat{k})}_{\text{Point } (\vec{a}_2)} + \mu \underbrace{(2\hat{i} + 3\hat{j} + \hat{k})}_{\text{Direction } (\vec{b}_2)}$$

Skew lines

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$d = \left| \frac{(3\hat{i} + 3\hat{j} + 3\hat{k}) \cdot (-9\hat{i} + 3\hat{j} + 9\hat{k})}{3\sqrt{19}} \right|$$

$$d = \left| \frac{-27 + 9 + 27}{3\sqrt{19}} \right|$$

$$d = \frac{3}{\sqrt{19}}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} \rightarrow$$

$$= \hat{i}(-9) - \hat{j}(-3) + \hat{k}(9) \\ = -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{81 + 9 + 81}$$

$$= \sqrt{171}$$

$$= \sqrt{9 \times 19}$$

$$= 3\sqrt{19}$$

Q.17) Find the shortest distance b/w the lines whose vector equations are $\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$

& $\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$.

$L_1: \vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$

$L_1: \vec{r} = (\hat{i} - 2\hat{j} + 3\hat{k}) + t(-\hat{i} + \hat{j} - 2\hat{k})$
 Point $(\vec{a}_1)$ Direction $(\vec{b}_1)$

Similarly

$L_2: \vec{r} = (\hat{i} - \hat{j} - \hat{k}) + s(\hat{i} + 2\hat{j} - 2\hat{k})$
 Point $(\vec{a}_2)$ Direction $(\vec{b}_2)$

Skew lines,

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$d = \left| \frac{(\hat{j} - 4\hat{k}) \cdot (2\hat{i} - 4\hat{j} - 3\hat{k})}{\sqrt{29}} \right|$$

$$d = \left| \frac{-4 + 12}{\sqrt{29}} \right|$$

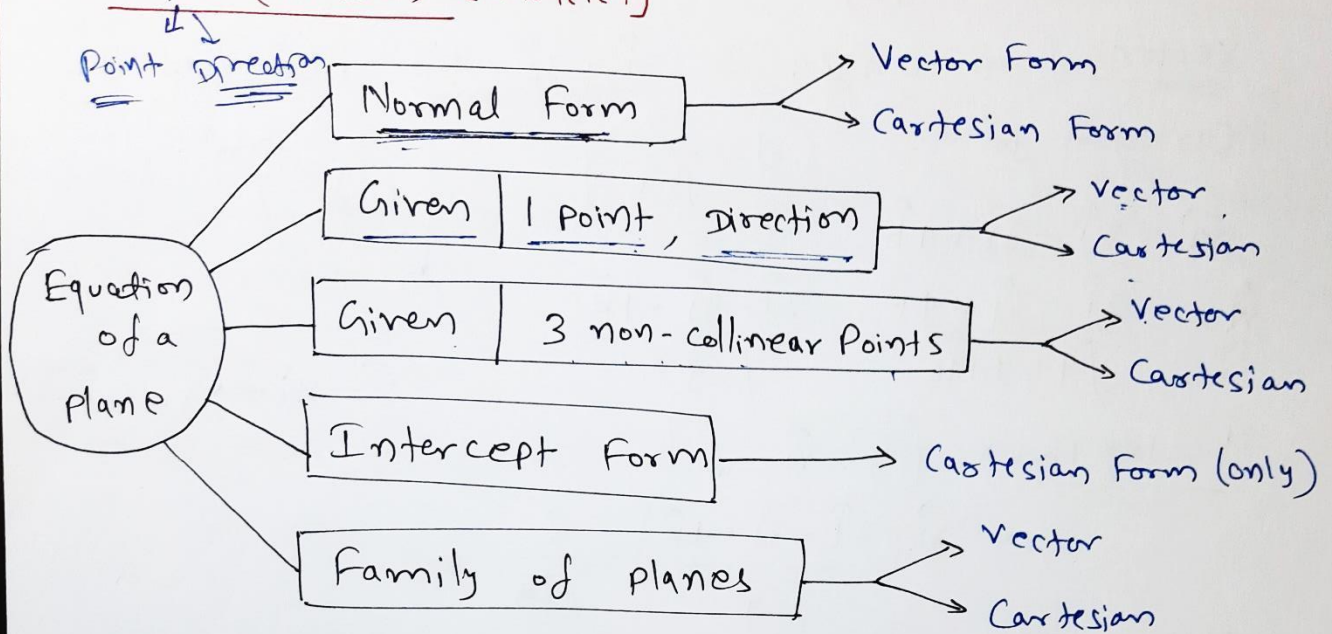
$$d = \frac{8}{\sqrt{29}}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= \hat{i}(2) - \hat{j}(4) + \hat{k}(-3)$$

$$= 2\hat{i} - 4\hat{j} - 3\hat{k}$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{4 + 16 + 9} = \sqrt{29}$$

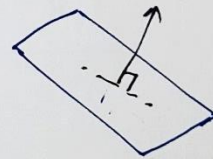
Plane (in 3D) [समतल]

Note: Line में Direction का मतलब



(Line के along या parallel vector की Direction)

Plane में Direction का मतलब



(Plane के perpendicular vector की Direction)

Normal Form of a Plane

Plane के perpendicular unit vector = $\hat{n}$
(given)

Plane की origin से Distance = d
(given)

Vector Form

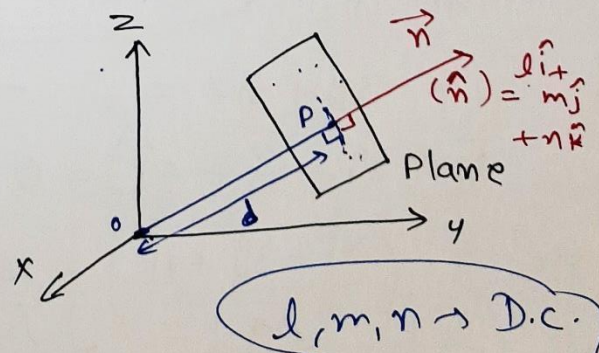
$$\vec{r} \cdot \hat{n} = d$$

variable = $x\hat{i} + y\hat{j} + z\hat{k}$

Cartesian Form

$$lx + my + nz = d$$

$$(l^2 + m^2 + n^2 = 1)$$



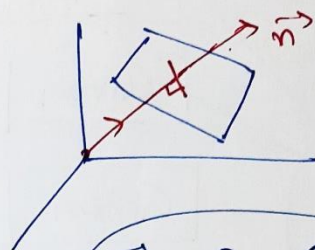
$$\vec{r} \cdot \hat{n} = d$$

e.g. Find the vector equation of a plane which is at a distance of $\frac{6}{\sqrt{29}}$ from the origin and its normal vector from origin is $2\hat{i} - 3\hat{j} + 4\hat{k}$. Also find its Cartesian form.

$$d = \frac{6}{\sqrt{29}}$$

$$\vec{n} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\hat{n} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{4+9+16}} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}$$



$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Vector form, $\vec{r} \cdot \hat{n} = d$

$$\Rightarrow \vec{r} \cdot \left(\frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}} \right) = \frac{6}{\sqrt{29}} \quad \checkmark$$

Cartesian form,

$$\frac{2x}{\sqrt{29}} - \frac{3y}{\sqrt{29}} + \frac{4z}{\sqrt{29}} = \frac{6}{\sqrt{29}} \quad \checkmark$$

e.g. Find the distance of the plane $2x - 3y + 4z - 6 = 0$ from origin. Also find D.R. & D.C. of vector perpendicular to it.

$$\text{D.R.} = 2, -3, 4$$

$$2x - 3y + 4z = 6 \quad \Rightarrow \quad \frac{2x - 3y + 4z}{\sqrt{29}} = \frac{6}{\sqrt{29}}$$

$$\vec{r} \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 6$$

$$\Rightarrow \vec{r} \cdot \left(\frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}} \right) = \frac{6}{\sqrt{29}}$$

$$\vec{r} \cdot \hat{n} = d$$

$$\text{Distance from origin} = \frac{6}{\sqrt{29}}$$

$$\frac{2x}{\sqrt{29}} - \frac{3y}{\sqrt{29}} + \frac{4z}{\sqrt{29}} = \frac{6}{\sqrt{29}}$$

$$\text{D.C.} = \frac{2}{\sqrt{29}}, -\frac{3}{\sqrt{29}}, \frac{4}{\sqrt{29}}$$

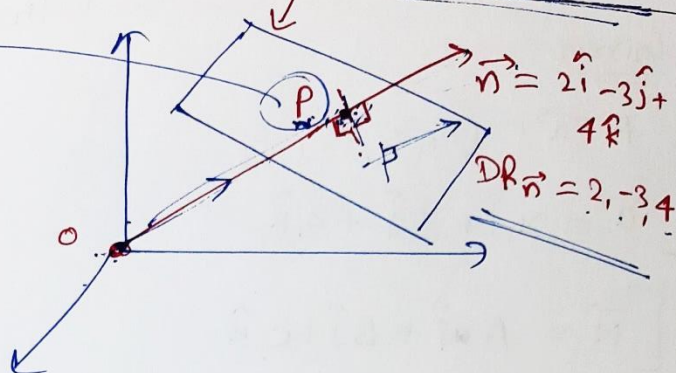
e.g. Find the coordinate of the foot of the perpendicular drawn ~~to~~ from the origin to the plane $2x - 3y + 4z - 6 = 0$

Ans: let $P(x_1, y_1, z_1) = ?$

$O(0, 0, 0)$

$\vec{OP} \rightarrow$ Direction Ratios

$DR_{OP} = (x_1, y_1, z_1)$



Plane $(2x - 3y + 4z - 6 = 0)$ $\rightarrow$ Perpendicular vector $(\vec{n})$
 $\rightarrow$ DR. $= (2, -3, 4)$

$$\vec{OP} \parallel \vec{n} \rightarrow \begin{matrix} DR_{OP} & \propto & DR_{\vec{n}} \\ \downarrow & & \downarrow \\ x_1, y_1, z_1 & & 2, -3, 4 \end{matrix}$$

$$\Rightarrow \frac{x_1}{2} = \frac{y_1}{-3} = \frac{z_1}{4} = \lambda$$

$$\boxed{x_1 = 2\lambda} \quad \boxed{y_1 = -3\lambda} \quad \boxed{z_1 = 4\lambda}$$

$\therefore$ Point 'P' (x_1, y_1, z_1) lies on the plane also.

$$2x - 3y + 4z - 6 = 0$$

$$\Rightarrow 2(x_1) - 3y_1 + 4z_1 - 6 = 0$$

$$\Rightarrow 2(2\lambda) - 3(-3\lambda) + 4(4\lambda) - 6 = 0$$

$$\Rightarrow 4\lambda + 9\lambda + 16\lambda = 6$$

$$\Rightarrow 29\lambda = 6$$

$$\lambda = \frac{6}{29}$$

$$P(x_1, y_1, z_1)$$

$$P(2\lambda, -3\lambda, 4\lambda)$$

$$P\left(\frac{12}{29}, -\frac{18}{29}, \frac{24}{29}\right)$$

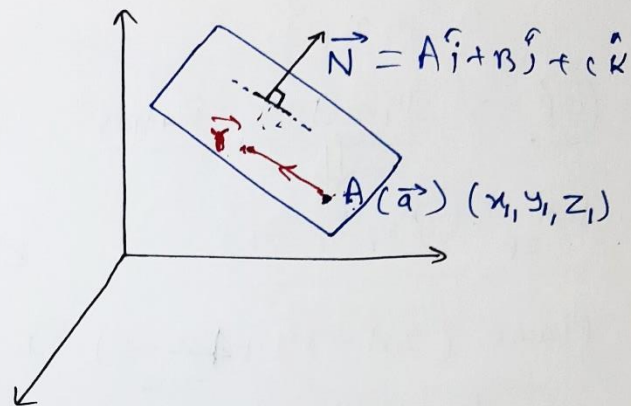
Equation of a plane (Given - 1 Point, Direction)

(Plane passes through this) (Perpendicular to plane)

Given

$$\begin{cases} A(\vec{a}) (x_1, y_1, z_1) \\ \vec{a} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} \end{cases}$$

$$\vec{N} = A\hat{i} + B\hat{j} + C\hat{k}$$



Plane:

Vector Form

$$(\vec{r} - \vec{a}) \cdot \vec{N} = 0$$

Point

Direction

(Perpendicular)

Cartesian Form

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

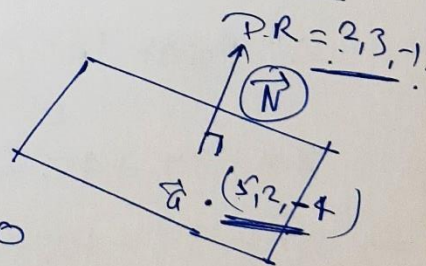
E.g. Find the vector and Cartesian eqⁿ of the plane which passes through the point $(5, 2, -4)$ and perpendicular to the line with direction ratios 2, 3, -1.

Cartesian Form:

$$\begin{aligned} 2(x-5) + 3(y-2) - 1(z+4) &= 0 \\ \Rightarrow 2x + 3y - z &= 20 \end{aligned}$$

Vector Form: $(\vec{r} - \vec{a}) \cdot \vec{N} = 0$

$$\Rightarrow (\vec{r} - 5\hat{i} - 2\hat{j} + 4\hat{k}) \cdot (2\hat{i} + 3\hat{j} - \hat{k}) = 0$$



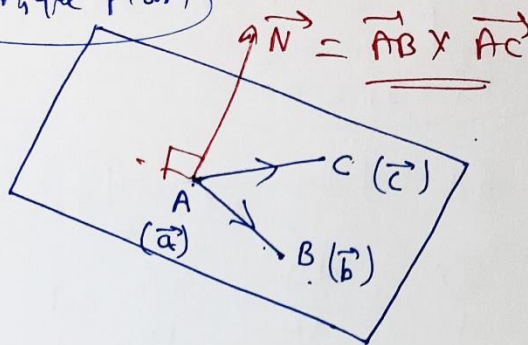
Equation of a plane

Given

3 non-collinear ~~plane~~ points

Points	Position Vector	Coordinates
A	$\vec{a}$	(x_1, y_1, z_1)
B	$\vec{b}$	(x_2, y_2, z_2)
C	$\vec{c}$	(x_3, y_3, z_3)

Unique Plane



Plane

Vector Form

$$(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$$

Cartesian Form

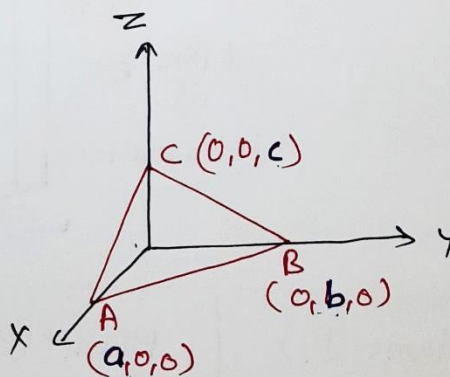
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Intercept Form

x-intercept = a

y-intercept = b

z-intercept = c



Equation of plane in Intercept form

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

e.g. Find x, y and z -intercepts of the plane

$$\vec{r} \cdot (6\hat{i} + 4\hat{j} + 3\hat{k}) = 12.$$

$$x\hat{i} + y\hat{j} + z\hat{k}$$

$$\Rightarrow 6x + 4y + 3z = 12$$

$$x\text{-intercept} = 2$$

$$\Rightarrow \frac{6x}{12} + \frac{4y}{12} + \frac{3z}{12} = 1$$

$$y \text{ ————— } = 3$$

$$z \text{ ————— } = 4$$

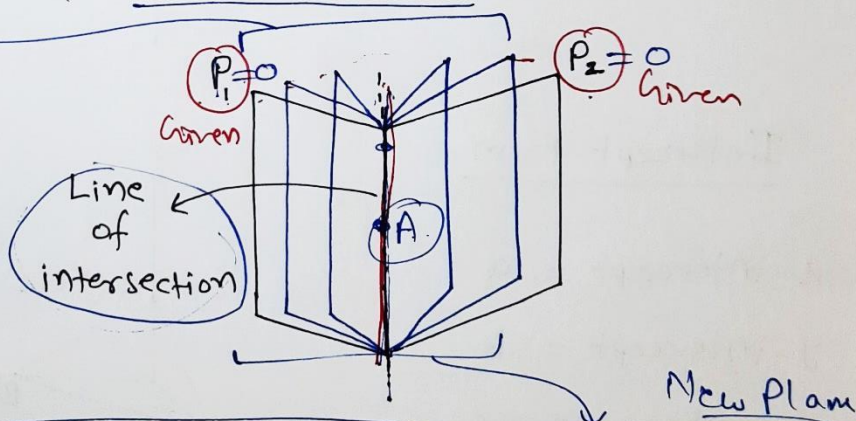
$$\Rightarrow \boxed{\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1}$$

Family of Planes

New (plane passing through the intersection of two given planes)

New Plane

$$\boxed{P_1 + \lambda P_2 = 0}$$



	Plane (P_1)	Plane (P_2)	Family of planes ($P_1 + \lambda P_2 = 0$)
Vector Form	$\vec{r} \cdot \vec{n}_1 = d_1$	$\vec{r} \cdot \vec{n}_2 = d_2$	$(\vec{r} \cdot \vec{n}_1 - d_1) + \lambda(\vec{r} \cdot \vec{n}_2 - d_2) = 0$
Cartesian Form	$A_1x + B_1y + C_1z = d_1$	$A_2x + B_2y + C_2z = d_2$	$(A_1x + B_1y + C_1z - d_1) + \lambda(A_2x + B_2y + C_2z - d_2) = 0$

e.g. Find the Cartesian equation of the plane New passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$, and also passes through origin.

New plane $(P_3) = 0$

$$\Rightarrow P_1 + \lambda(P_2) = 0$$

$$\Rightarrow (x+y+z-6) + \lambda(2x+3y+4z+5) = 0$$

$(0,0,0)$ satisfies —

$$\Rightarrow (0-6) + \lambda(0+5) = 0$$

$$\Rightarrow \boxed{\lambda = \frac{6}{5}}$$

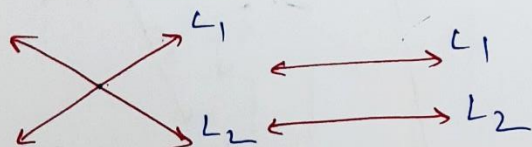
$$\begin{cases} P_1 \equiv x+y+z-6=0 \\ P_2 \equiv 2x+3y+4z+5=0 \end{cases}$$

New plane

$$(x+y+z-6) + \frac{6}{5}(2x+3y+4z+5) = 0$$

Coplanarity of two lines

Coplanar Lines



Yes, Point of intersection

Parallel

$$L_1: \vec{r} = \vec{a}_1 + \lambda \vec{b}_1$$

$$L_2: \vec{r} = \vec{a}_2 + \mu \vec{b}_2$$

$$d=0$$

Point

Vector Form

direction

Condition of coplanarity of two lines

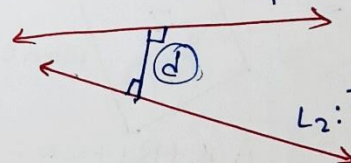
$$\boxed{(a_2 - a_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0}$$

Non-Coplanar

Skew lines

$$L_1: \vec{r} = \vec{a}_1 + \lambda \vec{b}_1$$

$$L_2: \vec{r} = \vec{a}_2 + \mu \vec{b}_2$$



- No point of intersection
- not parallel

$$d = \left| \frac{(a_2 - a_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right| \neq 0$$

$$d \neq 0$$

e.g. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and

$\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$

$$\vec{b}_1 = -3\hat{i} + \hat{j} + 5\hat{k}$$

$$\vec{b}_2 = -\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\vec{a}_1 = -3\hat{i} + \hat{j} + 5\hat{k}$$

$$\vec{a}_2 = -\hat{i} + 2\hat{j} + 5\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} + 0\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$$

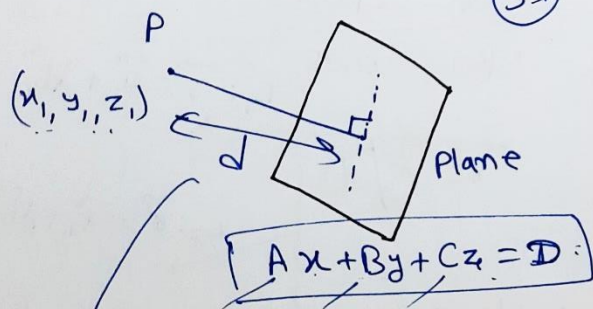
$$\vec{b}_1 \times \vec{b}_2 = -5\hat{i} + 10\hat{j} - 5\hat{k}$$

Condⁿ. of Coplanarity.

$$(2\hat{i} + \hat{j}) \cdot (-5\hat{i} + 10\hat{j} - 5\hat{k}) = -10 + 10 = 0$$

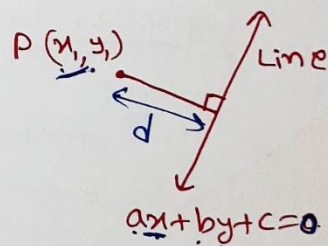
Distance of a Point from a Plane.

(3D)



$$d = \left| \frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2 + B^2 + C^2}} \right|$$

(2D)



$$d = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

e.g. Find the distance b/w Point $(2, 5, -3)$ and plane

$$\vec{r} \cdot (6\hat{i} - 3\hat{j} + 2\hat{k}) = 4$$

$$d = \left| \frac{12 - 15 - 6 - 4}{\sqrt{6^2 + (-3)^2 + 2^2}} \right|$$

$$= \left| \frac{-13}{7} \right| = \frac{13}{7}$$

$$6x - 3y + 2z - 4 = 0$$

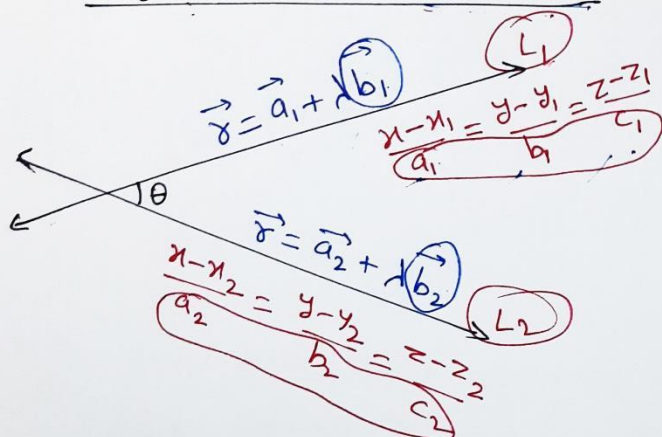
Angle between &

Dot Product
 $\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$

Line	Line	$\cos \theta$
Plane	Plane	$\cos \theta$
Plane	Line	<u>SMD</u> ?

Already Covered

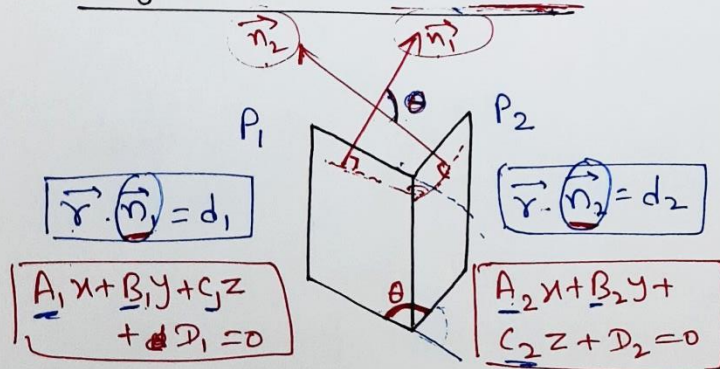
I Angle between Two Lines



$$\cos \theta = \left| \frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1| |\vec{b}_2|} \right|$$

$$\cos \theta = \left| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

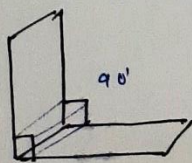
II Angle between Two Planes



$$\cos \theta = \left| \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|} \right|$$

$$\cos \theta = \left| \frac{A_1 A_2 + B_1 B_2 + C_1 C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

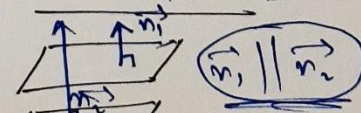
Note: ~~Perp~~ Perpendicular Planes



$\theta = 90^\circ$
 $\cos \theta = 0$

$$A_1 A_2 + B_1 B_2 + C_1 C_2 = 0$$

Parallel Planes



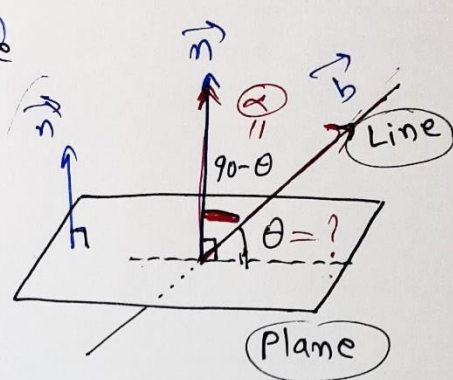
DR $\rightarrow$ Proportional

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

III Angle between a line & a plane

Line $\vec{r} = \vec{a} + \lambda \vec{b}$

Plane, $\vec{r} \cdot \vec{n} = d$



$$\cos(90 - \theta) = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}| |\vec{n}|}$$

$$\Rightarrow \sin \theta = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}| |\vec{n}|}$$

e.g. Find the angle between the planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$. (P_1) (P_2)

$$\cos \theta = \frac{|(2 \times 3) + 1 \times (-6) + (-2) \times (-2)|}{\sqrt{2^2 + 1^2 + (-2)^2} \sqrt{3^2 + (-6)^2 + (-2)^2}} = \frac{|6 - 6 + 4|}{3 \times 7}$$

$$\cos \theta = \frac{4}{21} \Rightarrow \theta = \cos^{-1}\left(\frac{4}{21}\right)$$

e.g. Find the angle between the line $\frac{x+1}{2} = \frac{y}{3} = \frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

Line direction, D.R. 2, 3, 6 ($\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$)

Plane direction, D.R. 10, 2, -11 ($\vec{n} = 10\hat{i} + 2\hat{j} - 11\hat{k}$)

$$\sin \theta = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}| |\vec{n}|} = \frac{(2\hat{i} + 3\hat{j} + 6\hat{k}) \cdot (10\hat{i} + 2\hat{j} - 11\hat{k})}{\sqrt{2^2 + 3^2 + 6^2} \sqrt{10^2 + 2^2 + (-11)^2}}$$

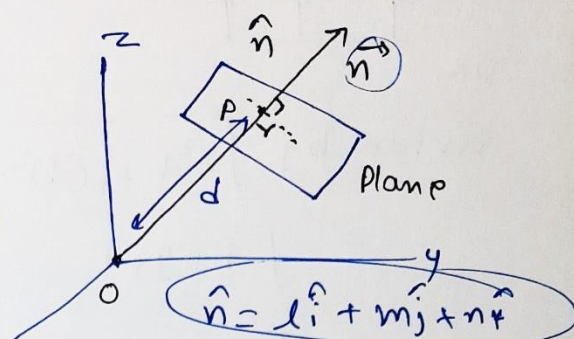
$$\sin \theta = \frac{|20 + 6 - 66|}{7 \times 15} \Rightarrow \sin \theta = \frac{40}{7 \times 15} \Rightarrow \theta = \sin^{-1}\left(\frac{8}{21}\right)$$

Exercise 11.3Plane (3D)

Eq. of plane,

$$\vec{r} \cdot \hat{n} = d$$

Vector Form

Direction
✓Distance
from originUnit vector
Perpendicular
to the plane

$$l^2 + m^2 + n^2 = 1$$

$$lx + my + nz = d$$

Cartesian Form.

D.C.

Distance from
origin

e.g. $Ax + By + Cz = d$

DR. $\rightarrow (A, B, C)$

D.C. $\left(\frac{A}{\sqrt{A^2+B^2+C^2}}, \frac{B}{\sqrt{A^2+B^2+C^2}}, \frac{C}{\sqrt{A^2+B^2+C^2}} \right)$

Exercise 11.3

Q.1 In each of the following cases, determine the direction cosines of the normal to the plane and the distance from the origin.

(a) $z = 2$

$$\Rightarrow 0x + 0y + 1z = 2$$

D.C. $(l=0, m=0, n=1)$

$$l^2 + m^2 + n^2 = 1$$

Distance from
origin = 2

(b) $x + y + z = 1$

DR. $(1, 1, 1)$

Divide by $\sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$
on both sides,

$$\Rightarrow \frac{x}{\sqrt{3}} + \frac{y}{\sqrt{3}} + \frac{z}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

D.C. $= \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Distance from (0,0,0)
 $= \frac{1}{\sqrt{3}}$

$$\textcircled{C} \quad 2x + 3y - z = 5$$

DR.

Divide by $\sqrt{2^2 + 3^2 + (-1)^2}$

$$= \sqrt{4 + 9 + 1}$$

$$= \sqrt{14} \text{ in LHS \& RHS}$$

Eqⁿ $\left[\frac{2x}{\sqrt{14}} + \frac{3y}{\sqrt{14}} - \frac{z}{\sqrt{14}} = \frac{5}{\sqrt{14}} \right]$

D.C. $\frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}, -\frac{1}{\sqrt{14}}$

$$d = \frac{5}{\sqrt{14}}$$

$$\textcircled{D} \quad 5y + 8 = 0$$

$$\Rightarrow 5y = -8$$

$$\Rightarrow -5y = 8$$

$$\Rightarrow 0.x - 5.y + 0.z = 8$$

DR.

Divide by $\sqrt{0^2 + (-5)^2 + 0^2}$

$$= \sqrt{25} = 5$$

$$\Rightarrow \frac{0.x}{5} - \frac{5y}{5} + \frac{0.z}{5} = \frac{8}{5}$$

D.C. $0, -1, 0$

$$d = \frac{8}{5}$$

Q.2 Find the vector equation of a plane which is at a distance 7 units from the origin and normal to the vector $3\hat{i} + 5\hat{j} - 6\hat{k}$.

$d = 7$

$$\vec{r} \cdot \hat{n} = d$$

unit vector

$$\Rightarrow \left[\vec{r} \cdot \left(\frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{70}} \right) = 7 \right]$$

normal vector ($\vec{n}$)

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|} = \frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{9 + 25 + 36}}$$

$$= \frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{70}}$$

$$= \frac{3\hat{i} + 5\hat{j} - 6\hat{k}}{\sqrt{70}}$$

Q.3 Find the Cartesian ~~And~~ equation of the following planes:

(a) $\vec{r} \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$ Vector Form.
 $(\vec{r} = x\hat{i} + y\hat{j} + z\hat{k})$ put.
 $\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} - \hat{k}) = 2$
 $\Rightarrow \boxed{x + y - z = 2}$
Cartesian Form,

(b) $\vec{r} \cdot (2\hat{i} + 3\hat{j} - 4\hat{k}) = 1$
Cartesian Form,
 $\boxed{2x + 3y - 4z = 1}$

(c) $\vec{r} \cdot [(s-2t)\hat{i} + (3-t)\hat{j} + (2s+t)\hat{k}] = 5$ Vector Form
Cartesian form, $\boxed{(s-2t)x + (3-t)y + (2s+t)z = 5}$

~~**Q.4** In the following cases, find the coordinates~~

Exercise 11.3

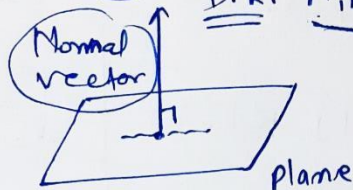
Plane (3D)

Plane

$$[\vec{r} \cdot \vec{n} = d]$$

Vector form

$$A\hat{i} + B\hat{j} + C\hat{k} = \vec{n}$$

D.R. A, B, C 

$$Ax + By + Cz = d$$

Cartesian form

Q.4 In the following cases, find the coordinates of the foot of perpendicular drawn from the origin.

(i) $2x + 3y + 4z - 12 = 0$

Foot of perpendicular drawn from origin = $P(x_1, y_1, z_1)$ (let)

$$\vec{n} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$DR_{\vec{n}} = 2, 3, 4$$

$$DR_{\vec{OP}} = x_1, y_1, z_1$$

$$\vec{n} \parallel \vec{OP} \rightarrow DR \rightarrow (\text{proportional})$$

$$\frac{x_1}{2} = \frac{y_1}{3} = \frac{z_1}{4} = \lambda \quad (\text{let})$$

$$P(x_1, y_1, z_1) ?$$

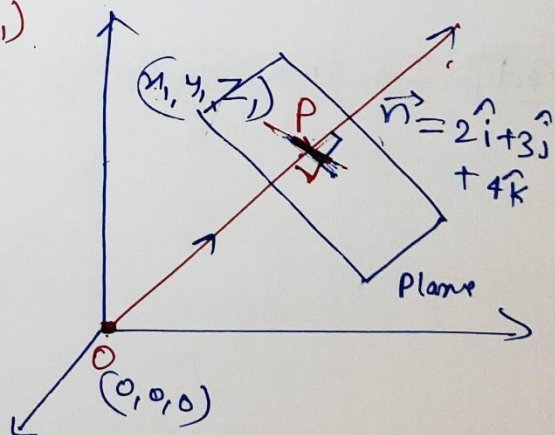
$$P(2\lambda, 3\lambda, 4\lambda)$$

$\uparrow \quad \uparrow \quad \uparrow$
 $x \quad y \quad z$

$$x_1 = 2\lambda, y_1 = 3\lambda, z_1 = 4\lambda$$

$$A(x_1, y_1, z_1) \rightarrow B(x_2, y_2, z_2)$$

$$DR_{AB} = x_2 - x_1, y_2 - y_1, z_2 - z_1$$



Plane $2x + 3y + 4z - 12 = 0$

Point 'P' lies on plane also.

Foot of Perpendicular

$P(x_1, y_1, z_1)$

or

$P(2\lambda, 3\lambda, 4\lambda)$

$\Rightarrow$ Point 'P' $(2\lambda, 3\lambda, 4\lambda)$ will satisfy the eqⁿ of plane $\boxed{2x + 3y + 4z - 12 = 0}$

$\Rightarrow 2(2\lambda) + 3(3\lambda) + 4(4\lambda) - 12 = 0$

$\Rightarrow 4\lambda + 9\lambda + 16\lambda = 12 \Rightarrow 29\lambda = 12 \Rightarrow \lambda = \frac{12}{29}$

Foot of Perpendicular drawn from origin

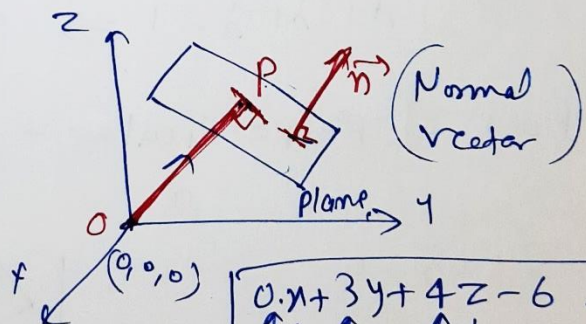
$P(x_1, y_1, z_1) \equiv P(2\lambda, 3\lambda, 4\lambda)$

$\equiv P\left(\frac{24}{29}, \frac{36}{29}, \frac{48}{29}\right)$

(b) $\boxed{3y + 4z - 6 = 0}$

Foot of Perpendicular

$P(x_1, y_1, z_1)$



$\boxed{0x + 3y + 4z - 6 = 0}$

$\vec{OR}_{OP} = x_1, y_1, z_1$

$\vec{OR}_{\vec{n}} = 0, 3, 4$

$\therefore \vec{OP} \parallel \vec{n} \rightarrow \vec{OR} \rightarrow \text{Proportional} \quad \frac{x_1}{0} = \frac{y_1}{3} = \frac{z_1}{4} = \lambda$ (let)

$P(0, 3\lambda, 4\lambda)$ satisfy the

eqⁿ of plane.

$3y + 4z - 6 = 0$

$\Rightarrow 3(3\lambda) + 4(4\lambda) - 6 = 0 \Rightarrow 25\lambda = 6 \Rightarrow \lambda = \frac{6}{25}$

Foot of perpendicular $P(0, 3\lambda, 4\lambda)$

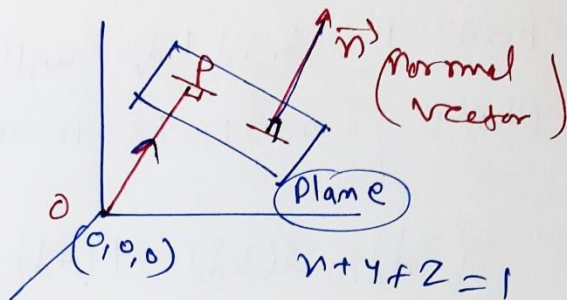
$$\lambda = \frac{6}{25}$$

$$\therefore P\left(0, \frac{18}{25}, \frac{24}{25}\right)$$

(C) $x+y+z=1$

Foot of perpendicular

$$P(x_1, y_1, z_1)$$



$$DR \vec{OP} = x_1, y_1, z_1$$

$$DR \vec{n} = 1, 1, 1$$

$$\vec{OP} \parallel \vec{n} \rightarrow DR = \text{Proportional} \rightarrow \frac{x_1}{1} = \frac{y_1}{1} = \frac{z_1}{1} = \lambda \text{ (let)}$$

$$P(x_1, y_1, z_1) = P(\lambda, \lambda, \lambda)$$

Point $P(\lambda, \lambda, \lambda)$ lies on $(x+y+z=1)$ plane.

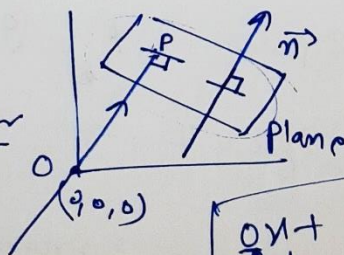
$$\Rightarrow \lambda + \lambda + \lambda = 1 \Rightarrow 3\lambda = 1 \Rightarrow \lambda = \frac{1}{3}$$

$$\text{Foot of perpendicular} = P\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$$

(d) $5y+8=0$

$P \rightarrow$ foot of perpendicular

$$(x_1, y_1, z_1)$$



$$DR \vec{OP} = x_1, y_1, z_1$$

$$DR \vec{n} = 0, 5, 0$$

$$\vec{OP} \parallel \vec{n} \rightarrow DR \rightarrow \text{Proportional} \rightarrow \frac{x_1}{0} = \frac{y_1}{5} = \frac{z_1}{0} = \lambda$$

$$x_1 = 0, y_1 = 5\lambda, z_1 = 0$$

Foot of Perpendicular $P(x, y, z)$
or $P(0, 5\lambda, 0)$

$$5\lambda = 8 \times \frac{-8}{2 \times 5}$$

Plane $5y + 8 = 0$

Satisfy,

$$\Rightarrow 5(5\lambda) + 8 = 0$$

$$\Rightarrow 25\lambda = -8$$

$$\Rightarrow \lambda = \frac{-8}{25}$$

$$P\left(0, -\frac{8}{5}, 0\right)$$

[Q.5] Find the Vector & Cartesian Eqⁿ of the Planes

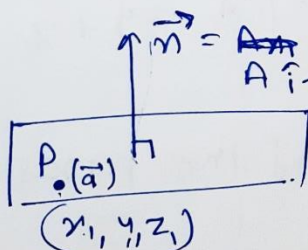
(a) that passes through the point $(1, 0, -2)$ and the normal to the plane is $\hat{i} + \hat{j} - \hat{k}$.

Standard form,

normal vector $(\vec{n})$

Vector form

$$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$$



$$D R(A, B, C)$$

Cartesian form,

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

Point $(1, 0, -2)$

$$\vec{a} = \hat{i} - 2\hat{k}$$

$$\vec{n} = \hat{i} + \hat{j} - \hat{k} \quad (R \Rightarrow 1, 1, -1)$$

Vector form,

$$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$$

$$[\vec{r} - (\hat{i} - 2\hat{k})] \cdot (\hat{i} + \hat{j} - \hat{k}) = 0$$

Cartesian form,

$$1(x - 1) + 1(y - 0) - 1(z + 2) = 0$$

$$\Rightarrow x - 1 + y - z - 2 = 0$$

$$\Rightarrow x + y - z = 3$$

(b) that passes through the point $(1, 4, 6)$ and the normal vector to the plane is $\hat{i} - 2\hat{j} + \hat{k}$.

Point $(1, 4, 6)$
 $\vec{a} = \hat{i} + 4\hat{j} + 6\hat{k}$

Normal vector

$$\vec{n} = \hat{i} - 2\hat{j} + \hat{k}$$

(D.R. = $\frac{1}{A}, \frac{-2}{B}, \frac{1}{C}$)

Vector Form:

$$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$$

$$\Rightarrow [\vec{r} - (\hat{i} + 4\hat{j} + 6\hat{k})] \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$$

Cartesian Form

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

$$\Rightarrow 1(x - 1) + (-2)(y - 4) + 1(z - 6) = 0$$

$$\Rightarrow (x - 1) - 2y + 8 + z - 6 = 0$$

$$\Rightarrow x - 2y + z - 1 = 0$$

Q.6 Find the equations of the planes that pass through three points.

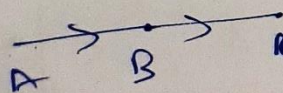
(a) $(1, 1, -1)$, $(6, 4, -5)$, $(-4, -2, 3)$
 $\begin{matrix} A & B & C \end{matrix} \rightarrow$



$$\left. \begin{array}{l} DR_{AB} = 5, 3, -4 \\ DR_{BC} = -10, -6, 8 \end{array} \right\} \frac{5}{-10} = \frac{3}{-6} = \frac{-4}{8}$$

(D.R. $\rightarrow$ Proportional)

$$-\frac{1}{2} = -\frac{1}{2} = -\frac{1}{2}$$



$\rightarrow$ Collinear $\rightarrow$

0 no. of planes are possible

(b) $\underset{A}{(1,1,0)}, \underset{B}{(1,2,1)}, \underset{C}{(-2,2,-1)}$

$\Rightarrow R_{AB} = 0, 1, -1$ } $\frac{0}{-3} \neq \frac{1}{0} \neq \frac{-1}{-2}$

$\Rightarrow R_{BC} = -3, 0, -2$

A, B, C $\rightarrow$ non-collinear points.

Unique Plane ✓

Eqn. of plane

A (1,1,0)
B (1,2,1)
C (-2,2,-1)

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-1 & z-0 \\ 1-1 & 2-1 & 1-0 \\ -2-1 & 2-1 & -1-0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-1 & z \\ 0 & 1 & 1 \\ -3 & 1 & -1 \end{vmatrix} = 0$$

$$\Rightarrow (x-1)[-1-1] - (y-1)(3) + z(3) = 0$$

$$\Rightarrow (x-1)(-2) - 3y + 3 \neq 3z = 0$$

$$\Rightarrow -2x + 2 - 3y + 3 + 3z = 0$$

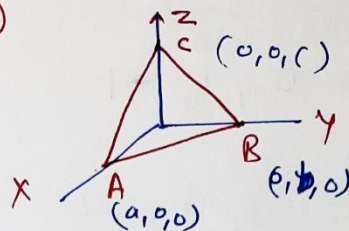
$$\Rightarrow \boxed{5 = 2x + 3y - 3z}$$

Exercise 11.3

Plane (3D)

Intercept Form.

$$\left[\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \right]$$



[Q.7] Find the intercepts cut off by the plane

$$2x + y - z = 5.$$

$$\Rightarrow \frac{2x}{5} + \frac{y}{5} - \frac{z}{5} = 1$$

$$\Rightarrow \left[\frac{x}{(5/2)} + \frac{y}{(5)} + \frac{z}{(-5)} = 1 \right]$$

$$x\text{-intercept} = \frac{5}{2} \checkmark$$

$$y\text{-intercept} = 5 \checkmark$$

$$z\text{-intercept} = -5 \checkmark$$

[Q.8] Find the equation of the plane with intercept 3 on y-axis and parallel to ZOX plane.

$$y\text{-intercept} = 3 = b$$

Intercept Form.

$$\left[\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \right]$$

$$\Rightarrow \left[\frac{x}{\infty} + \frac{y}{3} + \frac{z}{\infty} = 1 \right]$$

x-intercept

 ∞

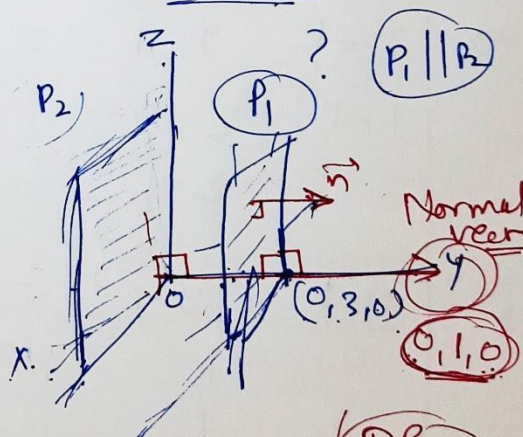
z-intercept

 ∞

$$\Rightarrow \frac{x}{\infty} + \frac{y}{3} + \frac{z}{\infty} = 1$$

$$\Rightarrow 0 + \frac{y}{3} + 0 = 1$$

$$\Rightarrow \boxed{y = 3}$$

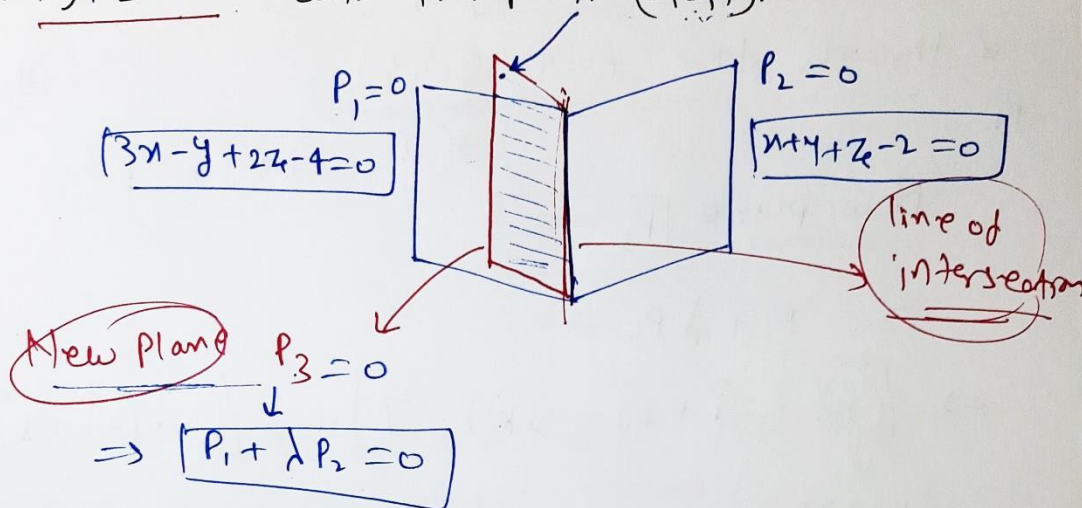


DR

$$\frac{\frac{1}{a}, \frac{1}{3}, \frac{1}{c}}{0, 1, 0}$$

$$\frac{\frac{1}{a}}{0} = \frac{\frac{1}{3}}{1} = \frac{1}{c} = 0$$

Q.9 Find the equation of the plane passing through the intersection of the planes $3x - y + 2z - 4 = 0$ and $x + y + z - 2 = 0$ and the point $(2, 2, 1)$.



$$\Rightarrow (3x - y + 2z - 4) + \lambda(x + y + z - 2) = 0$$

Also this plane passes through $(2, 2, 1)$

$$\Rightarrow (6 - 2 + 2 - 4) + \lambda(2 + 2 + 1 - 2) = 0$$

$$\Rightarrow 2 + \lambda(3) = 0$$

$$\Rightarrow \lambda = -\frac{2}{3}$$

Required plane: $(3x - y + 2z - 4) - \frac{2}{3}(x + y + z - 2) = 0$

$$\Rightarrow \frac{9x - 3y + 6z - 12 - 2x - 2y - 2z + 4}{3} = 0$$

$$\Rightarrow 7x - 5y + 4z - 8 = 0$$

Q.10 Find the vector equation of the plane passing through the intersection of the planes

$$\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) = 7 \quad \& \quad \vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) = 9 \quad \text{and}$$

through the point $(2, 1, 3)$.

P_1

P_2

$$\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$$

New plane $P_3 = 0$

$$\Rightarrow P_1 + \lambda P_2 = 0$$

$$\Rightarrow \left[\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) - 7 \right] + \lambda \left[\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) - 9 \right] = 0$$

variable variable

(this plane passes through the point $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$)

$$\Rightarrow (2\hat{i} + \hat{j} + 3\hat{k}) \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) - 7 + \lambda \left[(2\hat{i} + \hat{j} + 3\hat{k}) \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) - 9 \right] = 0$$

$$\Rightarrow 4 + 2 - 9 - 7 + \lambda [4 + 5 + 9 - 9] = 0$$

$$\Rightarrow \lambda(9) = 10$$

$$\Rightarrow \lambda = \frac{10}{9}$$

Required plane $\left[\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) - 7 \right]$

$$+ \left(\frac{10}{9} \right) \left[\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) - 9 \right] = 0$$

$$\Rightarrow \frac{9[\vec{r} \cdot (2\hat{i} + 2\hat{j} - 3\hat{k}) - 7] + 10[\vec{r} \cdot (2\hat{i} + 5\hat{j} + 3\hat{k}) - 9]}{9} = 0$$

$$\Rightarrow \vec{r} \cdot (18\hat{i} + 18\hat{j} - 27\hat{k} + 20\hat{i} + 50\hat{j} + 30\hat{k}) - 63 - 90 = 0$$

$$\Rightarrow \vec{r} \cdot (38\hat{i} + 68\hat{j} + 3\hat{k}) = 153$$

[Q.11] Find the eqⁿ. of the plane through the line of intersection of the planes $P_1: x+y+z=1$ and $P_2: 2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$.

Ans: $P_1: x+y+z-1=0$

$P_2: 2x+3y+4z-5=0$

New plane $P_3=0$

$$\Rightarrow P_1 + \lambda P_2 = 0$$

$$\Rightarrow (x+y+z-1) + \lambda(2x+3y+4z-5) = 0$$

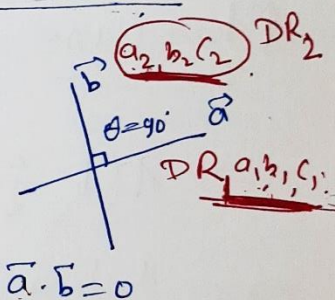
$$\Rightarrow x(1+2\lambda) + y(1+3\lambda) + z(1+4\lambda) - 1 - 5\lambda = 0$$

New plane

DR₁: $1+2\lambda, 1+3\lambda, 1+4\lambda$

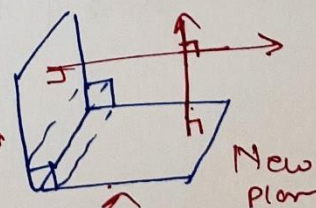
$x-y+z=0$

DR₂: $1, -1, 1$



$$\vec{a} \cdot \vec{b} = 0$$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$



New plane

New Plane $\vec{DR} = \underline{1+2\lambda, 1+3\lambda, 1+4\lambda}$

$$(x-y+z)_{=0} \vec{DR} = \underline{1, -1, 1}$$

Condition of Perpendicular Planes

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$\Rightarrow (1+2\lambda) - (1+3\lambda) + (1+4\lambda) = 0$$

$$\Rightarrow 1+2\lambda - 1-3\lambda + 1+4\lambda = 0$$

$$\Rightarrow 3\lambda + 1 = 0$$

$$\Rightarrow \lambda = -\frac{1}{3}$$

New plane $x(1+2\lambda) + y(1+3\lambda) + z(1+4\lambda) - 1 - 5\lambda = 0$

$$\Rightarrow x\left(1 - \frac{2}{3}\right) + y\left(1 - \frac{3}{3}\right) + z\left(1 - \frac{4}{3}\right) - 1 + \frac{5}{3} = 0$$

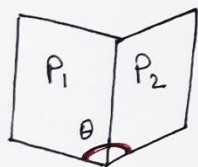
$$\Rightarrow x \cdot \frac{1}{3} + 0 + z\left(-\frac{1}{3}\right) + \frac{2}{3} = 0$$

$$\Rightarrow \frac{x}{3} - \frac{z}{3} + \frac{2}{3} = 0$$

$$\Rightarrow \boxed{x - z + 2 = 0}$$

Exercise 11.3

Plane (3D)



Angle b/w Two Planes

Vector Form

$$\begin{cases} P_1: \vec{r} \cdot \vec{n}_1 = d_1 \\ P_2: \vec{r} \cdot \vec{n}_2 = d_2 \end{cases}$$

$$\cos \theta = \left| \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|} \right|$$

Cartesian Form

$$P_1: A_1x + B_1y + C_1z = d_1$$

$$P_2: A_2x + B_2y + C_2z = d_2$$

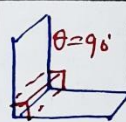
$$\cos \theta = \left| \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \cdot \sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

Condition for Parallel planes



$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

Condition for Perpendicular Planes



$$A_1A_2 + B_1B_2 + C_1C_2 = 0$$

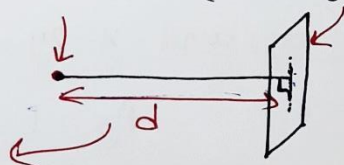
Distance Formula

[b/w Point & Plane]

(x₁, y₁, z₁)

(Ax + By + Cz = d)

$$d = \left| \frac{Ax_1 + By_1 + Cz_1 - d}{\sqrt{A^2 + B^2 + C^2}} \right|$$



Q.12 Find the angle b/w the planes whose vector equations are
 $\vec{r} \cdot (\underbrace{2\hat{i} + 2\hat{j} - 3\hat{k}}_{\vec{n}_1}) = 5$ and $\vec{r} \cdot (\underbrace{3\hat{i} - 3\hat{j} + 5\hat{k}}_{\vec{n}_2}) = 3$.

$$\cos \theta = \left| \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|} \right| = \left| \frac{(2\hat{i} + 2\hat{j} - 3\hat{k}) \cdot (3\hat{i} - 3\hat{j} + 5\hat{k})}{\sqrt{4+4+9} \cdot \sqrt{9+9+25}} \right|$$

$$\Rightarrow \cos \theta = \left| \frac{6 - 6 - 15}{\sqrt{17} \sqrt{43}} \right| \Rightarrow \cos \theta = \frac{15}{\sqrt{731}} \Rightarrow \theta = \cos^{-1} \left(\frac{15}{\sqrt{731}} \right)$$

Q.13 In the following cases, determine whether the given planes are parallel or perpendicular, and in case they are neither, find angles b/w them.

(a) $\underline{7x + 5y + 6z + 30 = 0}$ & $\underline{3x - y - 10z + 4 = 0}$

Condⁿ. for Parallel Planes $\frac{7}{3} \neq \frac{5}{-1} \neq \frac{6}{-10}$ Not Parallel -

Condⁿ. for Perpendicular Planes $21 - 5 - 60 \neq 0$
 $\Rightarrow 21 - 65 \neq 0$

Not $\perp$ planes

$$\cos \theta = \left| \frac{21 - 5 - 60}{\sqrt{49 + 25 + 36} \sqrt{9 + 1 + 100}} \right| = \left| \frac{44}{\sqrt{110} \sqrt{110}} \right|$$

$$\cos \theta = \frac{44}{110} \Rightarrow \cos \theta = \frac{2}{5} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{5}\right)$$

(b) $\underline{2x + y + 3z - 2 = 0}$ and $\underline{x - 2y + 5 = 0}$

$\underline{A_1 = 2}, \underline{B_1 = 1}, \underline{C_1 = 3}$

$\underline{A_2 = 1}, \underline{B_2 = -2}, \underline{C_2 = 0}$

Condⁿ. for Parallel Planes $\frac{2}{1} \neq \frac{1}{-2} \neq \frac{3}{0}$ Not Parallel

Condition for Perpendicular Planes

$$2 \cdot 1 - 2 + 0 = 0$$

$$\boxed{0 = 0} \checkmark$$

Perpendicular planes

(c) $2x - 2y + 4z + 5 = 0$ and $3x - 3y + 6z - 1 = 0$

Condition for Parallel planes

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} \checkmark$$

$$\Rightarrow \frac{2}{3} = \frac{-2}{-3} = \frac{4}{6} \Rightarrow \frac{2}{3} = \frac{2}{3} = \frac{2}{3}$$

$\therefore$ Parallel planes.

(d) $2x - y + 3z - 1 = 0$ & $2x - y + 3z + 3 = 0$

Condⁿ. for Parallel planes.

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

$$\Rightarrow \frac{2}{2} = \frac{-1}{-1} = \frac{3}{3} \Rightarrow 1 = 1 = 1$$

$\therefore$ Parallel planes.

(e) $4x + 8y + z - 8 = 0$ and $y + z - 4 = 0$

Condⁿ. for Parallel planes.

$$\frac{4}{0} \neq \frac{8}{1} \neq \frac{1}{1} \quad \text{Not Parallel}$$

Condⁿ. for $\perp$ Planes.

$$A_1A_2 + B_1B_2 + C_1C_2 \neq 0$$

$$\Rightarrow 4 \cdot 0 + 8 \cdot 1 + 1 \cdot 1 \neq 0$$

$$\Rightarrow 0 + 8 + 1 \neq 0$$

Not $\perp$ planes

$$\cos \theta = \frac{0 + 8 + 1}{\sqrt{16 + 64 + 1} \sqrt{0 + 1 + 1}} \Rightarrow \cos \theta = \frac{9}{\sqrt{81} \sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\theta = 45^\circ$$

Q.14 In the following cases, find the distance of each of the given points from the corresponding given plane:

(a) Point $(0,0,0)$; plane : $3x - 4y + 12z = 3$

$3x - 4y + 12z - 3 = 0$ (x, y, z) $Ax + By + Cz = d$
 $\rightarrow Ax + By + Cz - d = 0$

$$d = \left| \frac{0 - 0 + 0 - 3}{\sqrt{9 + 16 + 144}} \right|$$

$$d = \left| \frac{-3}{\sqrt{169}} \right| \Rightarrow d = \frac{3}{13}$$

$$d = \left| \frac{Ax_1 + By_1 + Cz_1 - d}{\sqrt{A^2 + B^2 + C^2}} \right|$$

(b) Point $(3, -2, 1)$, plane $2x - y + 2z + 3 = 0$

$$d = \left| \frac{6 + 2 + 2 + 3}{\sqrt{4 + 1 + 4}} \right| = \frac{13}{\sqrt{9}} = \frac{13}{3}$$

(c) Point $(2, 3, -5)$ plane : $x + 2y - 2z = 9$
 $\Rightarrow x + 2y - 2z - 9 = 0$

$$d = \left| \frac{2 + 6 + 10 - 9}{\sqrt{1 + 4 + 4}} \right| = \frac{9}{\sqrt{9}} = \frac{9}{3} = 3$$

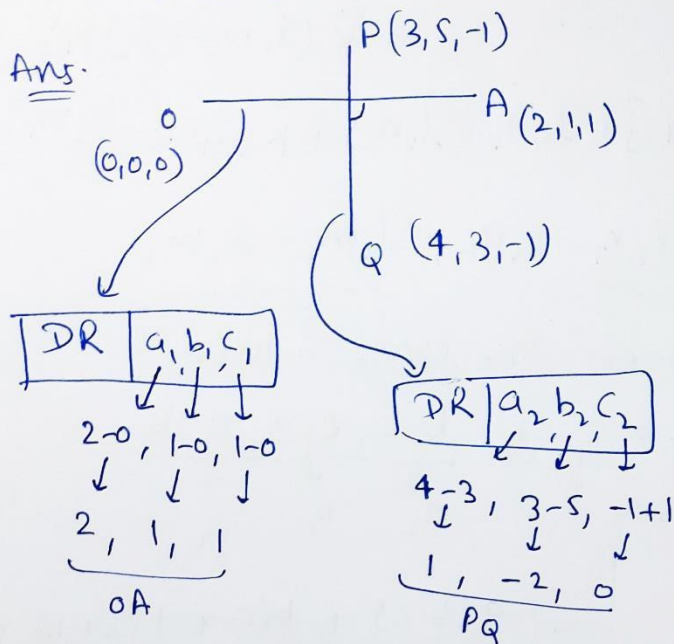
(d) Point $(-6, 0, 0)$ Plane : $2x - 3y + 6z - 2 = 0$

$$d = \left| \frac{-12 + 0 + 0 - 2}{\sqrt{4 + 9 + 36}} \right| = \left| \frac{-14}{\sqrt{49}} \right| = \frac{14}{7} = 2$$

Miscellaneous Exercise on Chapter 11

Q.1 Show that the line joining the origin to the point $A(2,1,1)$ is perpendicular to the line determined by the points $P(3,5,-1)$ & $Q(4,3,-1)$.

Ans.



Condition of Perpendicular lines

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$\begin{aligned} & a_1 a_2 + b_1 b_2 + c_1 c_2 \\ &= 2(1) + 1(-2) + 1(0) \\ &= 2 - 2 + 0 \\ &= 0 \end{aligned}$$

Perpendicular lines

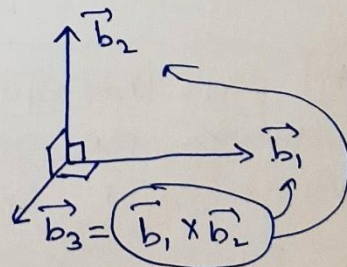
Q.2 If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1 n_2 - m_2 n_1, n_1 l_2 - n_2 l_1, l_1 m_2 - l_2 m_1$.

$$\vec{b}_1 = l_1 \hat{i} + m_1 \hat{j} + n_1 \hat{k}$$

$$(l_1^2 + m_1^2 + n_1^2 = 1) \rightarrow |\vec{b}_1| = 1$$

$$\vec{b}_2 = l_2 \hat{i} + m_2 \hat{j} + n_2 \hat{k}$$

$$(l_2^2 + m_2^2 + n_2^2 = 1) \rightarrow |\vec{b}_2| = 1$$



Now, $\vec{b}_3 = \vec{b}_1 \times \vec{b}_2$ will be perpendicular to both $\vec{b}_1$ & $\vec{b}_2$.

$$\vec{b}_3 = \vec{b}_1 \times \vec{b}_2 = (l_1\hat{i} + m_1\hat{j} + n_1\hat{k}) \times (l_2\hat{i} + m_2\hat{j} + n_2\hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = \hat{i}(m_1n_2 - m_2n_1) - \hat{j}(l_1n_2 - l_2n_1) + \hat{k}(l_1m_2 - l_2m_1)$$

$$= \hat{i}(m_1n_2 - m_2n_1) + \hat{j}(l_2n_1 - l_1n_2) + \hat{k}(l_1m_2 - l_2m_1)$$

D.C. $m_1n_2 - m_2n_1, l_2n_1 - l_1n_2, l_1m_2 - l_2m_1$

[Q.3] Find the angle between the lines whose direction ratios are a, b, c & $b-c, c-a, a-b$.

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} = \frac{a(b-c) + b(c-a) + c(a-b)}{\sqrt{a^2 + b^2 + c^2} \sqrt{(b-c)^2 + (c-a)^2 + (a-b)^2}}$$

$$\cos\theta = \frac{\cancel{ab} - \cancel{ac} + \cancel{bc} - \cancel{ba} + \cancel{ca} - \cancel{cb}}{\sqrt{\quad} \sqrt{\quad}} = \frac{0}{\sqrt{\quad} \sqrt{\quad}}$$

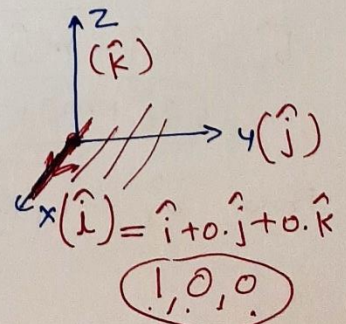
$\cos\theta = 0$ $\theta = 90^\circ$ Perpendicular

[Q.4] Find the equation of a line parallel to x-axis & passing through the origin.

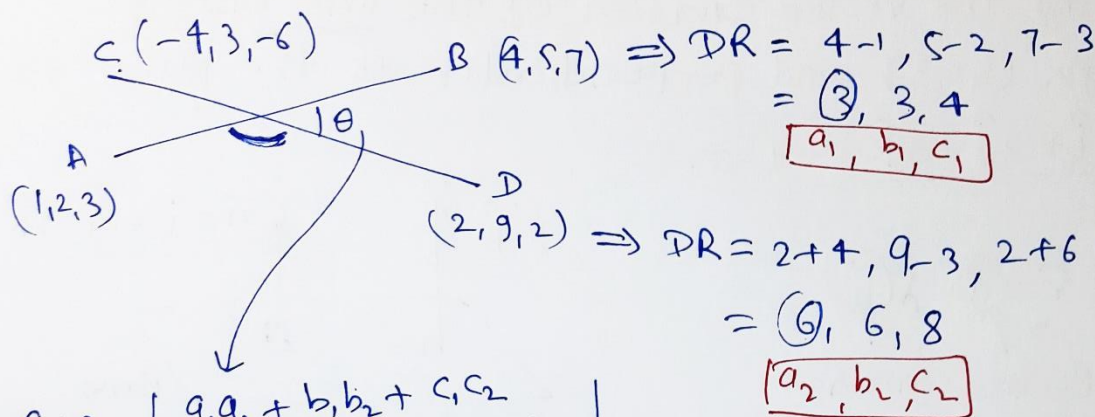
Point: $(0, 0, 0)$ Direction: $(1, 0, 0)$

Eqⁿ. of line $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

$$\Rightarrow \frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0} \Rightarrow \frac{x}{1} = \frac{y}{0} = \frac{z}{0}$$



[Q.5] If the coordinates of the points A, B, C, D are (1, 2, 3), (4, 5, 7), (-4, 3, -6) and (2, 9, 2) respectively, then find angle b/w the lines AB & CD.



$$\cos \theta = \left| \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}} \right|$$

$$= \left| \frac{3 \times 6 + 3 \times 6 + 4 \times 8}{\sqrt{9+9+16} \sqrt{36+36+64}} \right| = \left| \frac{18+18+32}{\sqrt{34} \sqrt{136}} \right|$$

$$\Rightarrow \cos \theta = \frac{68}{\sqrt{2 \times 17} \sqrt{4 \times 2 \times 17}} = \frac{68}{\sqrt{2 \times 17 \times 4 \times 2 \times 17}}$$

$$\Rightarrow \cos \theta = \frac{68}{2 \times 17 \times 2} = 1$$

$$\Rightarrow \cos \theta = 1 \Rightarrow \theta = 0^\circ$$

[Q.6] If the lines $\frac{x-1}{-3} = \frac{y-2}{2K} = \frac{z-3}{2}$ and $\frac{x-1}{3K} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find K.

Condⁿ. for $\perp$ lines:

$$a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$\Rightarrow -9K + 2K - 10 = 0 \Rightarrow -7K = 10 \Rightarrow K = \frac{10}{-7}$$

Miscellaneous Exercise on Chapter 11

[Q.7] Find the vector equation of the line passing through $(1, 2, 3)$ and perpendicular to the plane $\vec{r} \cdot (\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$.

Line:

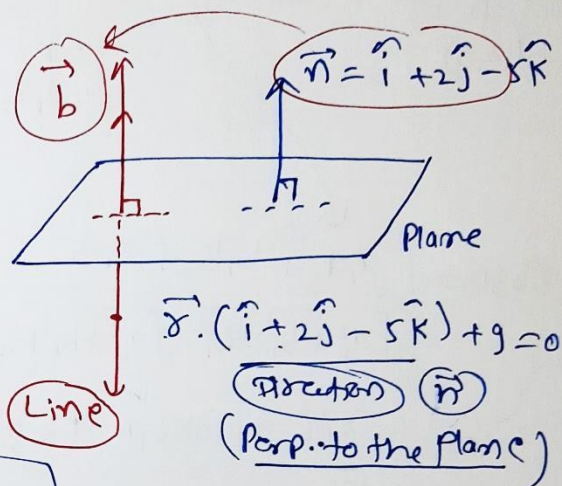
$$\vec{r} = \vec{a} + \lambda \vec{b}$$

Point $(1, 2, 3)$
 $(\hat{i} + 2\hat{j} + 3\hat{k})$

Direction $\vec{b} \parallel \vec{n}$

Line:

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda (\hat{i} + 2\hat{j} - 5\hat{k})$$



[Q.8] Find the equation of the plane passing through (a, b, c) and parallel to the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$.

Plane

$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$

$A = B = C = 1$

(A, B, C)
 $(1, 1, 1)$

DR $(1, 1, 1)$

$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 2$

$\vec{n}_2 = 1, 1, 1$

Required plane

$$1(x - a) + 1(y - b) + 1(z - c) = 0$$

$$\Rightarrow x + y + z = a + b + c$$

Q.9 Find the shortest distance b/w lines

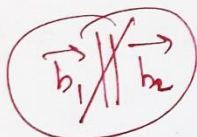
$$\vec{r} = (6\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}) \quad \& \quad \vec{r} = (-4\hat{i} - \hat{k}) + \mu(3\hat{i} - 2\hat{j} - 2\hat{k})$$

Point ($\vec{a}_1$)

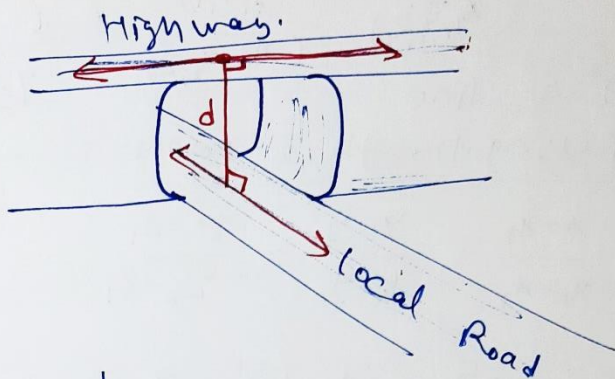
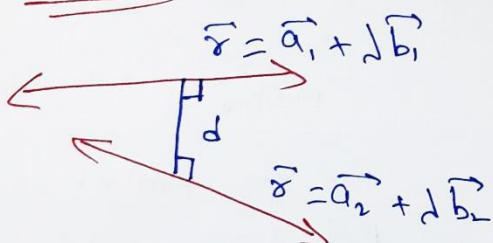
Direction ($\vec{b}_1$)

Point ($\vec{a}_2$)

Direction ($\vec{b}_2$)



Skew Lines



$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$d = \left| \frac{(-10\hat{i} - 2\hat{j} - 3\hat{k}) \cdot (8\hat{i} + 8\hat{j} + 4\hat{k})}{12} \right|$$

$$d = \left| \frac{-80 - 16 - 12}{12} \right|$$

$$d = \left| \frac{-108}{12} \right|$$

$$d = \frac{108}{12} = 9$$

$$d = 9$$

$$\vec{b}_1 \times \vec{b}_2$$

$$= (\hat{i} - 2\hat{j} + 2\hat{k}) \times (3\hat{i} - 2\hat{j} - 2\hat{k})$$

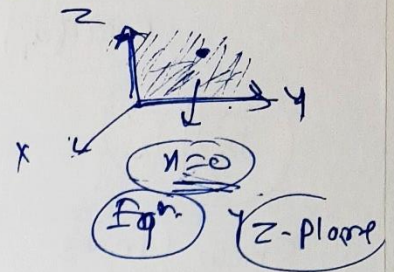
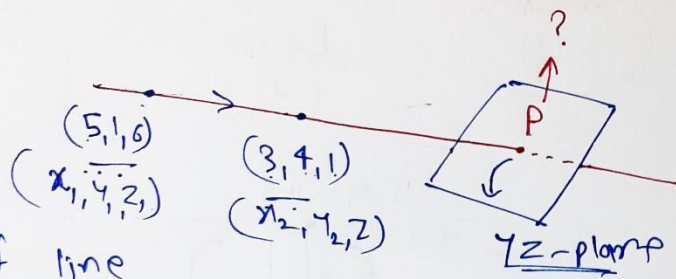
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} \rightarrow R_1$$

$$= \hat{i}(4 + 4) - \hat{j}(-2 - 6) + \hat{k}(-2 + 6)$$

$$= 8\hat{i} + 8\hat{j} + 4\hat{k} = \vec{b}_1 \times \vec{b}_2$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{64 + 64 + 16} = \sqrt{144} = 12$$

Q.10 Find the coordinates of the point where the line through (5,1,6) and (3,4,1) crosses the yz-plane.



Eqⁿ. of line
passing through 2-points.

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\Rightarrow \frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5} = \lambda$$

for 'P'

$$\frac{x-5}{-2} = \lambda \Rightarrow \boxed{x = -2\lambda + 5}$$

$$\frac{y-1}{3} = \lambda \Rightarrow \boxed{y = 3\lambda + 1}$$

$$\frac{z-6}{-5} = \lambda \Rightarrow \boxed{z = -5\lambda + 6}$$

Method-II

For yz-plane,

Put $x=0$

$$\frac{0-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5}$$

Point P $(-2\lambda + 5, 3\lambda + 1, -5\lambda + 6)$

Satisfies the plane

(yz-plane)

Eqⁿ. $\boxed{x=0}$

$$\Rightarrow -2\lambda + 5 = 0$$

$$\Rightarrow \boxed{\lambda = \frac{5}{2}}$$

Required

Point P $(-2\lambda + 5, 3\lambda + 1, -5\lambda + 6)$

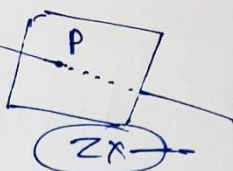
$$P\left[-2\left(\frac{5}{2}\right) + 5, 3\left(\frac{5}{2}\right) + 1, -5\left(\frac{5}{2}\right) + 6\right]$$

$$P\left(0, \frac{17}{2}, -\frac{13}{2}\right) \checkmark$$

Q.11 Find the coordinates of the point where the line through $(5, 1, 6)$ & $(3, 4, 1)$ crosses the Zx -plane.

Ans.

line eqⁿ → same as
Previous question.



$y = 0$ ①

By solving eqⁿ ① & ②

$$\frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5} \quad \text{--- ②}$$

$$\frac{x-5}{-2} = \frac{0-1}{3} = \frac{z-6}{-5}$$

$$\frac{x-5}{-2} = \frac{-1}{3} \Rightarrow x-5 = \frac{2}{3} \Rightarrow x = 5 + \frac{2}{3} \Rightarrow \boxed{x = \frac{17}{3}}$$

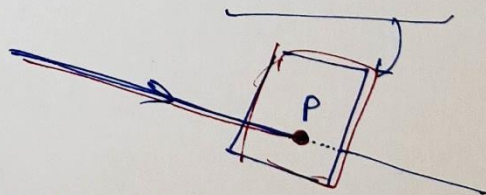
$$\frac{-1}{3} = \frac{z-6}{-5} \Rightarrow \frac{5}{3} + 6 = z \Rightarrow \boxed{\frac{23}{3} = z}$$

Point of intersection $P\left(\frac{17}{3}, 0, \frac{23}{3}\right)$

Q.12 Find the coordinates of the point where the line through $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane $2x + y + z = 7$.

Line: $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$

$$\begin{aligned} \Rightarrow \frac{x-3}{-1} &= \lambda & \left| \quad \frac{y+4}{1} &= \lambda \quad \right| & \begin{aligned} z+5 &= 6\lambda \\ z &= 6\lambda - 5 \end{aligned} \\ \Rightarrow \underline{x} &= \underline{-\lambda + 3} & \Rightarrow \underline{y} &= \underline{\lambda - 4} & \end{aligned}$$



$P(-\lambda + 3, \lambda - 4, 6\lambda - 5) \Rightarrow$ Also satisfies the plane

Point of intersection $P(-\lambda+3, \lambda-4, 6\lambda-5)$

Satisfies the plane $2x+y+z=7$.

$$\Rightarrow 2(-\lambda+3) + (\lambda-4) + (6\lambda-5) = 7$$

$$\Rightarrow -2\lambda+6 + \lambda-4 + 6\lambda-5 = 7$$

$$\Rightarrow 5\lambda = 10$$

$$\boxed{\lambda = 2}$$

$$P(-2+3, 2-4, 12-5)$$

$$\underline{\underline{P(1, -2, 7)}}$$

Miscellaneous Exercise on Chapter (11)

Q.13 Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to each of the planes $x+2y+3z=5$ and $3x+3y+z=0$

Required plane, (P) $A(x-x_1)+B(y-y_1)+C(z-z_1)=0$
 $(x_1, y_1, z_1) \equiv (-1, 3, 2)$

P: $A(x+1)+B(y-3)+C(z-2)=0$ $DR_p = A, B, C$

$P_1: x+2y+3z=5 \rightarrow DR_{P_1} = 1, 2, 3$ ($\vec{n}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$)

$P_2: 3x+3y+z=0 \rightarrow DR_{P_2} = 3, 3, 1$ ($\vec{n}_2 = 3\hat{i} + 3\hat{j} + \hat{k}$)

ATQ Condition for Perpendicular Planes.

$A_1A_2+B_1B_2+C_1C_2=0$

$P \perp P_1$

$Ax_1+Bx_2+Cx_3=0$

$\Rightarrow A+2B+3C=0$

$\Rightarrow A = -2B-3C$

$\Rightarrow A = -2\left(-\frac{8C}{3}\right) - 3C$

$\Rightarrow A = \frac{16C}{3} - 3C$

$\Rightarrow A = \frac{7C}{3}$ ✓

$P \perp P_2$

$3A+3B+C=0$

$\Rightarrow 3(-2B-3C)+3B+C=0$

$\Rightarrow -6B-9C+3B+C=0$

$\Rightarrow -3B-8C=0$

$\Rightarrow -3B=8C$

$\Rightarrow B = -\frac{8C}{3}$ ✓

Required plane $A(x+1) + B(y-3) + C(z-2) = 0$

$$A = \frac{7C}{3}, \quad B = \frac{-8C}{3}$$

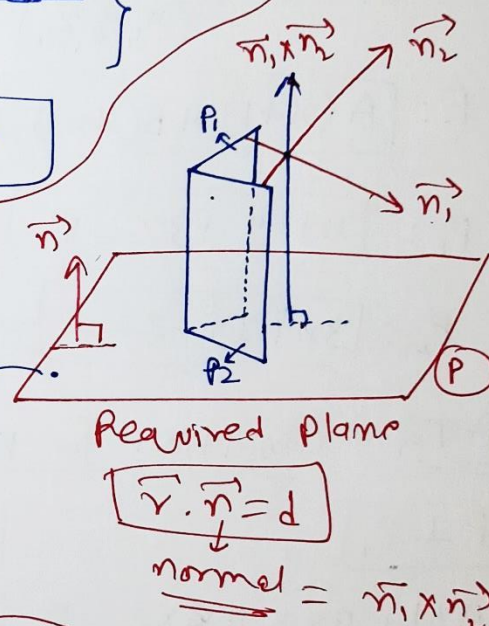
$$\Rightarrow \frac{7C}{3}(x+1) - \frac{8C}{3}(y-3) + C(z-2) = 0$$

$$\Rightarrow \textcircled{C} \left\{ \frac{7x+7-8y+24+3z-6}{3} \right\} = 0$$

$$\Rightarrow 7x - 8y + 3z + 25 = 0$$

II - method
(Hint only)
vector method

$(-1, 3, 2)$



Q.14 If the points $(1, 1, P)$ and $(-3, 0, 1)$ be equidistant from the ~~point~~ plane $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$, then find the value of P .

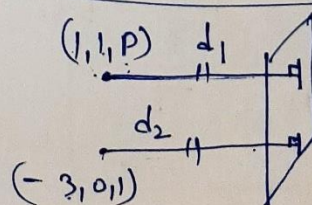
$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$3x + 4y - 12z + 13 = 0$$

(x_1, y_1, z_1)

$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$

$Ax + By + Cz = d$



ATQ. $d_1 = d_2$

$$\Rightarrow \left| \frac{3+4-12P+13}{\sqrt{9+16+144}} \right| = \left| \frac{-9+0-12+13}{\sqrt{9+16+144}} \right|$$

$$\Rightarrow \underline{\underline{|20-12P| = |-8| = 8}}$$

$$\Rightarrow \boxed{20-12P = \pm 8}$$

$$20-12P = 8$$

$$\Rightarrow 12 = 12P$$

$$\Rightarrow \boxed{P=1} \checkmark$$

$$20-12P = -8$$

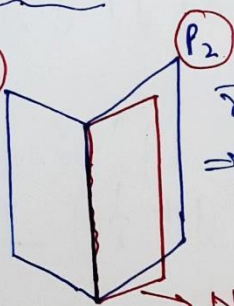
$$\Rightarrow 28 = 12P$$

$$\Rightarrow \boxed{\frac{7}{3} = P} \checkmark$$

Q.15 Find the equation of the plane passing through the line of intersection of the planes

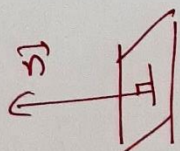
$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$ and parallel to x-axis.

$\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$ P_1
 $\Rightarrow x+y+z=1$
 $\Rightarrow \boxed{x+y+z-1=0}$ P_1



$\vec{r} \cdot (2\hat{i} + 3\hat{j} - \hat{k}) + 4 = 0$
 $\Rightarrow \boxed{2x+3y-z+4=0}$

New plane $\boxed{P_1 + \lambda P_2 = 0}$

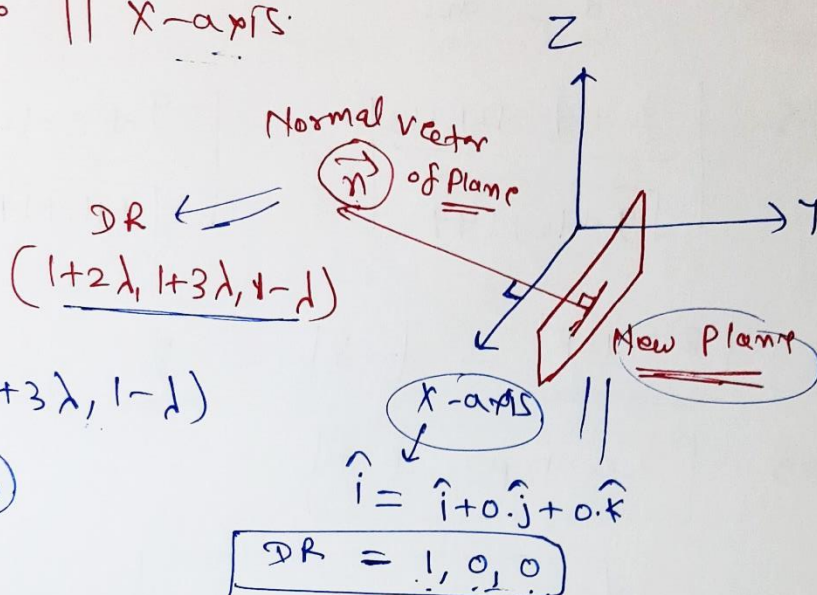


$$\Rightarrow (x+y+z-1) + \lambda(2x+3y-z+4) = 0$$

$$\Rightarrow \boxed{x(1+2\lambda) + y(1+3\lambda) + z(1-\lambda) + (4\lambda-1) = 0}$$

DR $\rightarrow \underline{1+2\lambda}, \underline{1+3\lambda}, \underline{1-\lambda}$

New plane || X-axis.



$$\vec{n} \rightarrow (1+2\lambda, 1+3\lambda, 1-\lambda)$$

$$\text{x-axis} \rightarrow (1, 0, 0)$$

By Diagram,

$\vec{n} \perp \text{x-axis}$

$$\text{Condition } a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$\Rightarrow (1+2\lambda) \cdot 1 + (1+3\lambda) \cdot 0 + (1-\lambda) \cdot 0 = 0$$

$$\Rightarrow 1+2\lambda = 0$$

$$\Rightarrow \lambda = -\frac{1}{2}$$

New plane,
$$(1+2\lambda)x + (1+3\lambda)y + (1-\lambda)z + (4\lambda-1) = 0$$

Put $\lambda = -\frac{1}{2}$

$$\Rightarrow 0 \cdot x + \left(1 - \frac{3}{2}\right)y + \left(1 + \frac{1}{2}\right)z + \left(\frac{-4}{2} - 1\right) = 0$$

$$\Rightarrow \left(-\frac{1}{2}y\right) + \left(\frac{3}{2}z\right) = 3$$

$$\Rightarrow \frac{-y + 3z}{2} = 3$$

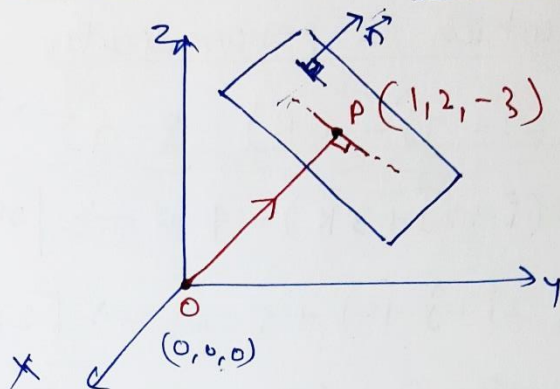
$$\Rightarrow -y + 3z = 6 \Rightarrow 0 = y - 3z + 6$$

[Q.16] If 'O' be the origin and the coordinates of 'P' be $P(1, 2, -3)$, then find the equation of the plane passing through P and perpendicular to OP.

$$O(0, 0, 0)$$

$$P(1, 2, -3)$$

$$DR_{OP} = 1, 2, -3$$



Any plane passing through the point (x_1, y_1, z_1)

$$Eq^n \Rightarrow A(x - x_1) + B(y - y_1) + C(z - z_1) = 0$$

$$P(1, 2, -3)$$

Plane, $A(x - 1) + B(y - 2) + C(z + 3) = 0$

$$DR = A, B, C \leftarrow \text{DR of normal vector of Plane}$$

By Diagram, $\vec{OP} \parallel \vec{n}$

DR $\rightarrow$	1, 2, -3	A, B, C
------------------	----------	---------

$$\underline{A=1}, \underline{B=2}, \underline{C=-3}$$

Required Plane, $1(x - 1) + 2(y - 2) + (-3)(z + 3) = 0$

$$\Rightarrow x - 1 + 2y - 4 - 3z - 9 = 0$$

$$\Rightarrow x + 2y - 3z = 14$$

[Q.17] Find the equation of the plane which contains the line of intersection of the planes $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0$, $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ and which is perpendicular to the plane

$$\vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0 \rightarrow P_3$$

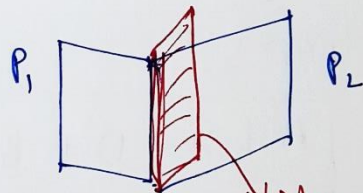
$$P_1: \vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) - 4 = 0 \Rightarrow \boxed{x + 2y + 3z - 4 = 0} P_1$$

$$P_2: \vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0 \Rightarrow \boxed{2x + y - z + 5 = 0} P_2$$

$$P_3: \vec{r} \cdot (5\hat{i} + 3\hat{j} - 6\hat{k}) + 8 = 0 \Rightarrow \boxed{5x + 3y - 6z + 8 = 0} P_3$$

New plane $\rightarrow$ passing through the line of intersection of P_1 & P_2

$$\boxed{P_1 + \lambda P_2 = 0}$$



$$\Rightarrow (x + 2y + 3z - 4) + \lambda(2x + y - z + 5) = 0$$

$$\Rightarrow \boxed{x(1 + 2\lambda) + y(2 + \lambda) + z(3 - \lambda) + (5\lambda - 4) = 0} \text{ New plane}$$

$\downarrow$ This New plane is perpendicular to the plane $P_3: \boxed{5x + 3y - 6z + 8 = 0}$

$$DR(1 + 2\lambda, 2 + \lambda, 3 - \lambda)$$

Condition of Perpendicular planes.

$$DR: \underline{5, 3, -6}$$

$$A_1 A_2 + B_1 B_2 + C_1 C_2 = 0$$

$$\Rightarrow (1 + 2\lambda) \cdot 5 + (2 + \lambda) \cdot 3 + (3 - \lambda) \cdot (-6) = 0$$

$$\Rightarrow \underline{5} + \underline{10\lambda} + \underline{6} + \underline{3\lambda} - \underline{18} + \underline{6\lambda} = 0$$

$$\Rightarrow 19\lambda = 7 \Rightarrow \boxed{\lambda = 7/19}$$

New plane : $x(1+2\lambda) + y(2+\lambda) + z(3-\lambda) + (5\lambda-4) = 0$

Put $(\lambda = \frac{7}{19}) \uparrow$

$$\Rightarrow x\left(1 + \frac{14}{19}\right) + y\left(2 + \frac{7}{19}\right) + z\left(3 - \frac{7}{19}\right) + \left(\frac{35}{19} - 4\right) = 0$$

$$\Rightarrow x\left(\frac{33}{19}\right) + y\left(\frac{45}{19}\right) + z\left(\frac{50}{19}\right) + \left(-\frac{41}{19}\right) = 0$$

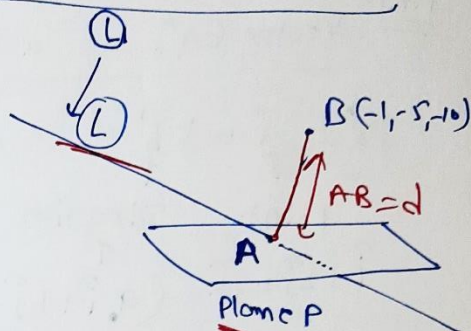
$$\Rightarrow \boxed{33x + 45y + 50z - 41 = 0}$$

Miscellaneous Exercise on Chapter 11

Q.18 Find the distance of the point $B(-1, -5, -10)$ from the point of intersection of the line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

Plane P

A $\rightarrow$ Point of intersection of line L & plane P



We solve the equations of line L & plane P for the point of intersection.

Line L: $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ — (1)

Plane P: $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ — (2)

Substitution method.

$$\Rightarrow [(2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})] \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$$

(Dot Product)

$$\Rightarrow 2 + 1 + 2 + \lambda(3 - 4 + 2) = 5$$

$$\Rightarrow 5 + \lambda(1) = 5$$

$$\Rightarrow \boxed{\lambda = 0}$$

By eqn (1):

$$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k}$$

Point of Intersection (A)

$$A(2, -1, 2)$$

$$AB = \sqrt{(3)^2 + 4^2 + 12^2} = \sqrt{9 + 16 + 144} = \sqrt{169} = 13$$

$B(-1, -5, -10)$ — given



Q.19 Find the vector equation of the line passing through $(1, 2, 3)$ and parallel to the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$.

Ans. Vector Eqⁿ of a line.

$$\vec{r} = \underset{\substack{\uparrow \\ \text{Point}}} {\vec{a}} + \lambda \underset{\substack{\uparrow \\ \text{Direction}}} {\vec{b}}$$

$$\hat{i} + 2\hat{j} + 3\hat{k} \quad (a\hat{i} + b\hat{j} + c\hat{k})$$

$$\Rightarrow \boxed{\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda (a\hat{i} + b\hat{j} + c\hat{k})} \quad \text{--- (B)}$$

Our required line is parallel to the plane (P_1)

$$\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$$

$$\Rightarrow \boxed{\vec{b} \cdot \vec{n}_1 = 0}$$

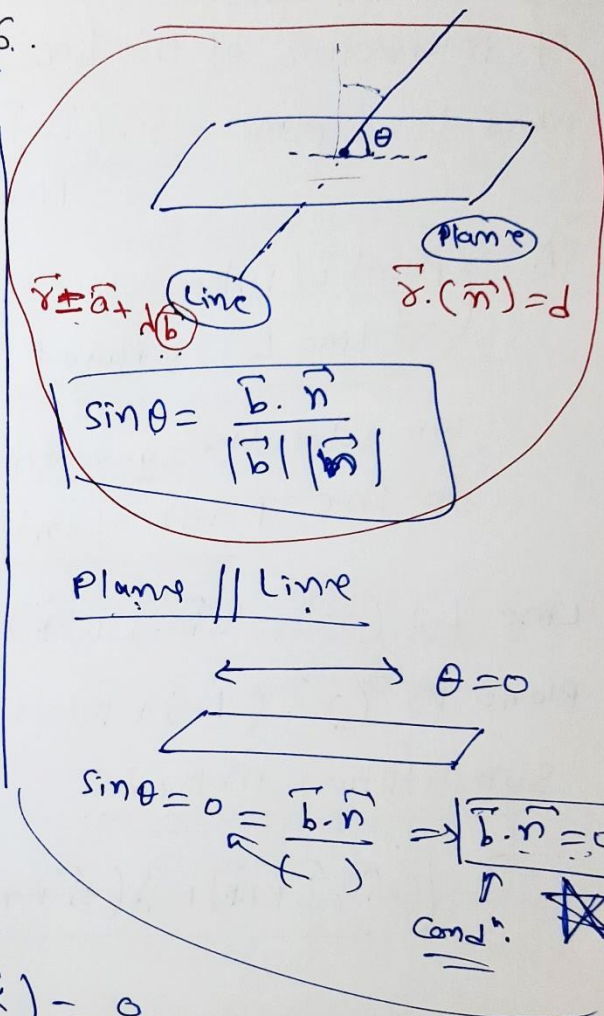
$$\Rightarrow (a\hat{i} + b\hat{j} + c\hat{k}) \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 0$$

$$\Rightarrow \boxed{a - b + 2c = 0} \quad \text{--- (1)}$$

Line L $\parallel$ $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$

$$\Rightarrow (a\hat{i} + b\hat{j} + c\hat{k}) \cdot (3\hat{i} + \hat{j} + \hat{k}) = 0$$

$$\Rightarrow \boxed{3a + b + c = 0} \quad \text{--- (2)}$$



By eqⁿ (1) & (2)

Elimination.

$$a - b + 2c = 0$$

$$3a + b + c = 0$$

$$+ \quad \hline 4a + 3c = 0$$

$$\boxed{c = -\frac{4a}{3}}$$

by putting the value of $c = -\frac{4a}{3}$ in eqn (1)

$$\Rightarrow a + 2c = b$$

$$\Rightarrow a + 2\left(-\frac{4a}{3}\right) = b$$

$$\Rightarrow \frac{3a - 8a}{3} = b \Rightarrow \boxed{b = -\frac{5a}{3}}$$

$$\boxed{a - b + 2c = 0}$$

Required Pqⁿ. of line $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(a\hat{i} + b\hat{j} + c\hat{k})$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\left(\frac{a}{1}\hat{i} - \frac{5a}{3}\hat{j} - \frac{4a}{3}\hat{k}\right)$$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda\left(\frac{3a\hat{i} - 5a\hat{j} - 4a\hat{k}}{3}\right)$$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \underbrace{\lambda\left(-\frac{a}{3}\right)}_{\lambda_1} (-3\hat{i} + 5\hat{j} + 4\hat{k})$$

$$\Rightarrow \boxed{\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda_1 (-3\hat{i} + 5\hat{j} + 4\hat{k})}$$

[Q.20] Find the vector equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

Vector Eqⁿ. of line (L)
 $\vec{r} = \vec{a} + \lambda \vec{b}$
 Point $\uparrow$ Direction $\uparrow$
 $(\hat{i} + \hat{j} - 4\hat{k})$ $(a\hat{i} + b\hat{j} + c\hat{k})$

Direction

$$\vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}$$

Concept

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$$

$$\vec{r} = \vec{a}_2 + \mu \vec{b}_2$$

$$\perp \rightarrow \boxed{\vec{b}_1 \cdot \vec{b}_2 = 0}$$

Direction

$$\vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$$

ATQ Line $L \perp L_1$
 $(\vec{b} = a\hat{i} + b\hat{j} + c\hat{k})$ $(\vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k})$

$$\Rightarrow \vec{b} \cdot \vec{b}_1 = 0$$

$$\Rightarrow \boxed{3a - 16b + 7c = 0} \quad \text{--- (1)}$$

ATQ, Line $L \perp L_2$
 $(\vec{b} = a\hat{i} + b\hat{j} + c\hat{k})$ $(\vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k})$

$$\Rightarrow \vec{b} \cdot \vec{b}_2 = 0$$

$$\Rightarrow \boxed{3a + 8b - 5c = 0} \quad \text{--- (2)}$$

By eqn. (1) - eqn. (2)

$$\begin{array}{r} 3a - 16b + 7c = 0 \\ 3a + 8b - 5c = 0 \\ \hline -24b + 12c = 0 \end{array}$$

$$\Rightarrow 12c = 24b$$

$$\boxed{c = 2b}$$

By eqn. (2)

$$\Rightarrow 3a + 8b - 5(2b) = 0$$

$$\Rightarrow 3a + 8b - 10b = 0$$

$$\Rightarrow 3a = 2b \Rightarrow \boxed{a = \frac{2b}{3}}$$

Required line $\vec{r} = \vec{a} + \lambda \vec{b}$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda (\frac{2b}{3}\hat{i} + b\hat{j} + 2b\hat{k})$$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda (\frac{2b}{3}\hat{i} + b\hat{j} + 2b\hat{k})$$

$$\Rightarrow \vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + b\lambda (\frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{2})$$

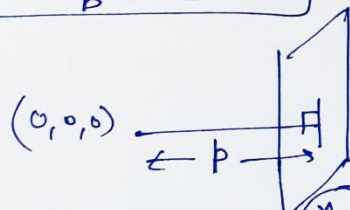
$$\Rightarrow \boxed{\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda_1 (2\hat{i} + 3\hat{j} + 6\hat{k})}$$

New λ_1

Q.21) Prove that if a plane has intercepts a, b, c and is at a distance ' p ' units from the origin, then $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{p^2}$.

Intercept form of the plane.

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

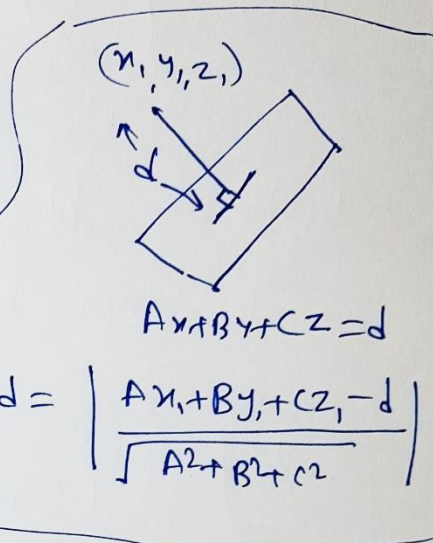


$$P = \frac{0+0+0-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

$$\Rightarrow P = \frac{1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

$$\Rightarrow \frac{1}{p} = \sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}$$



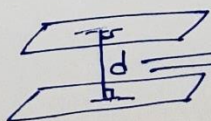
Q.22) Distance b/w the two planes $2x + 3y + 4z = 4$ (P₁) & $4x + 6y + 8z = 12$ (P₂) is —

- (A) 2 units (B) 4 (C) 8 (D) $\frac{2}{\sqrt{29}}$

$$P_1: 2x + 3y + 4z - 4 = 0$$

$$P_2: 4x + 6y + 8z - 12 = 0$$

A B C



I-method.

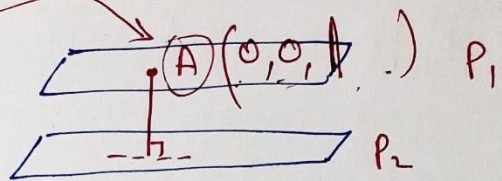
$$d = \frac{|d_1 - d_2|}{\sqrt{A^2 + B^2 + C^2}}$$

$$d = \frac{|(-4) - (-6)|}{\sqrt{2^2 + 3^2 + 4^2}} = \frac{2}{\sqrt{29}}$$

$$(4, 9, 16)$$

II - method
 $P_1: 2x + 3y + 4z - 4 = 0$

$P_2: 2x + 3y + 4z - 6 = 0$



Point A on 'P₁'
 $(0, 0, 1)$

Distance b/w P₁ & P₂

= Distance b/w A & P₂

(Point) (Plane)
 $(0, 0, 1)$

$2x + 3y + 4z - 6 = 0$

$$= \left| \frac{0 + 0 + 4 - 6}{\sqrt{2^2 + 3^2 + 4^2}} \right| = \frac{2}{\sqrt{29}}$$

let Point $(0, 0, z)$
 on 'P₁'
 $\Rightarrow 0 + 0 + 4z - 4 = 0$
 $\Rightarrow \boxed{z = 1}$

Q.23 The planes: $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$ $2x - y + 4z = 5$ and $5x - 2.5y + 10z = 6$
 are (A) perpendicular (B) Parallel
 (C) intersect y-axis (D) passes through $(0, 0, \frac{5}{4})$

Condition of Parallel planes

$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$

$\Rightarrow \frac{2}{5} = \frac{+10}{+2.5} = \frac{4}{10}$

$\Rightarrow \frac{2}{5} = \frac{+10}{\cancel{2.5}} = \frac{\cancel{4}2}{\cancel{10}5}$

$\frac{2}{5} = \frac{2}{5} = \frac{2}{5}$